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Optimizing charging schedules and assessing passenger commuting equilibrium for electric shuttles with shared charging stations

Yili Tang [a,b], Bissan Ghaddar [c,d], Jaehee Jeong [e,*], Yi Wei [a]

[a] Department of Civil and Environmental Engineering, University of Western Ontario, London, Ontario, Canada
[b] Department of Electrical and Computer Engineering, University of Western Ontario, London, Ontario, Canada
[c] Ivey Business School, University of Western Ontario, London, Ontario, Canada
[d] Science and Technology, IE University, Madrid, Spain
[e] Department of Industrial Engineering, Pusan National University, Busan, Republic of Korea

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ABSTRACT

Transit electrification has been rapidly growing to decarbonize our communities and form a cleaner and more advanced transit system for sustainable mobility. Transit is also the only transportation option for many communities, such as youth and the elderly. However, affordable and feasible charging infrastructure and operation remain a key challenge for electric transit due to the complex planning and associated costs. This study proposes an electric transit charging strategy with shared public charging stations rather than dedicated transit charging installation. The study devises an optimal charging schedule to minimize the charging costs and further evaluate the impacts of the charging schedules on passengers' onboarding patterns through a commuting user equilibrium model. The proposed optimization model of charging scheduling determines the number of service trips that involve charging processes, the selected public charging stations, and the timing of charges, thus making decisions about charging plans for transit vehicles. The corresponding passenger onboarding patterns show that transit vehicles with higher battery capacities and better conditions (e.g., newer versus older batteries) lead to lower individual equilibrium trip costs for passengers. The proposed strategy and models offer new forms of electric transit adoption and operations paving the way for sustainable pathways for transit operators and policy makers.

1. Introduction

The transportation industry is responsible for approximately 25% of global carbon dioxide emissions due to the widespread reliance on vehicles powered by carbon-based fuels (Javanmard et al., 2023). In this context, electric public bus (e-bus) systems are viewed as a crucial component in providing cost-effective and eco-friendly transportation options. However, the deployment of e-buses introduces significant operational challenges, particularly in the context of charging infrastructure, investment, and scheduling. Typical electric transit systems install depot charging stations for overnight charging, yet it requires long-term and high-cost investments. On the other hand, opportunity charging where buses charge during short layovers at designated stops has emerged as an alternative charging strategy (McCabe and Ban, 2023). Such systems have been tested and deployed in several European and U.S. cities (Ashkezari et al.,

* Corresponding author.
E-mail addresses: ytang564@uwo.ca (Y. Tang), bghaddar@ivey.ca (B. Ghaddar), jjaehee@pusan.ac.kr (J. Jeong), ywei445@uwo.ca (Y. Wei).

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2024). Although opportunity charging provides flexibility for charging locations, it maintains the key challenge of infrastructure costs and planning, particularly in installing dedicated facilities, selecting charger locations and determining the most suitable stops for installation.

While dedicated charging infrastructure for transit systems requires substantial investment and time-consuming planning, the rapid growth of public charging stations, especially fast charging stations for private vehicles, opens new opportunities for transit systems to access and share the existing charging infrastructure. In addition, electricity does not require physical storage space, which is different from fueling gasoline. The rapid growth of battery capacity and charging technologies also increases the performance of public charging stations for various electric transit vehicles including regular buses and shuttle buses. Especially for shuttle buses such as community buses or school buses, they have relatively small battery size and energy consumption, as well as more flexible and short-distance schedules which exhibit potential for the utilization with public charging. All of these shed light on the readiness of shared charging strategies between transit systems and public charging stations. Several studies explored opening depot charging stations to private electric vehicles to improve the return on the investments, energy utilization and emission reductions (Liu et al., 2024; Heliox, 2023). Discussions on challenges such as asset security, visitor fleet management and regulatory policy are also explored which requires long-term organisational frameworks and strategic planning (Cenext, 2025). On the contrary, this study explores transit charging at the public charging stations without necessities of depot charging investments and maintenance. Public charging stations provide the flexibility to charge e-buses at various points in their routes, potentially extending operational range without requiring significant detours.

To this end, this study is one of the first attempts to propose and investigate the strategy of e-buses using existing public charging stations which offers a cost-effective and flexible approach, leveraging established infrastructure to reduce upfront investments in dedicated charging depots. This strategy can facilitate the early adoption of e-buses, particularly for regions where dedicated charging infrastructure is still under development or with the deployment of community-based electric shuttle buses. The study also develops a passenger onboarding user equilibrium model to analyze the impacts of shared charging strategies on passengers' choices and onboarding patterns to provide user-centric electric transit pathways. Thus the contribution of this paper is threefold. First, we introduce a novel shared charging strategy for electric buses, offering a cost-effective and flexible alternative to dedicated charging depots. By leveraging existing infrastructure, this approach reduces upfront investment costs while ensuring efficient charging operations. Second, we develop a mixed-integer programming optimization model to determine the optimal charging locations and scheduling, followed by a passenger onboarding user equilibrium model to assess the impact of shared charging strategies on passenger choices and travel behavior. Third, we present a case study from London, Ontario, to demonstrate the practical implications of these methodologies. The results show that charging activities significantly influence passengers' departure time choices and onboarding patterns, where higher battery capacity with fewer charging trips can reduce queuing time and alleviate crowding during peak hours. We also provide insights into the impact of battery characteristics and pricing strategies by comparing scenarios involving new versus aged batteries, as well as time-varying versus fixed electricity pricing schemes.

The remainder of this paper is organized as follows: Section 2 reviews the literature on electric bus charging infrastructure and scheduling operations. Section 3 outlines the description and configuration of the problem. Sections 4 and 5 present the solution methodology, including the optimization model and the assessment of behavioral uncertainty. Section 6 provides a case study in which the proposed methodologies are tested on an e-shuttle service for commuting. Finally, Section 7 summarizes the findings and discusses future research directions.

2. Literature review

2.1. Electric transit initiatives and charging infrastructure

The transition to electric buses is a pivotal step in achieving sustainable urban transportation systems. As cities worldwide seek to reduce greenhouse gas emissions and combat air pollution, e-buses offer a viable solution by replacing conventional diesel-powered buses with clean and efficient alternatives (Manzolli et al., 2022). For instance, the Toronto Transit Commission (TTC) has committed to electrifying its bus fleet, with plans to procure 340 battery-electric buses by the end of 2026. This initiative aims to make e-buses approximately 20% of the TTC's total bus fleet, supporting the city's goal of achieving 50% zero-emissions by 2030 and 100% zero-emissions before 2040 (Electric Autonomy Canada, 2024). OC Transpo, Ottawa's transit agency, has initiated the transition to a fully electric bus fleet as well, with currently 4 e-buses and plans to add 26 new battery-electric buses by the end of 2025 and reach a total of 354 in the fleet by the end of 2027. The city aims to have a completely electric fleet by 2036, aligning with its environmental objectives (OC Transpo, 2024). Similarly, the City of Brampton is developing a zero emission bus implementation strategy and rollout plan to transition its transit system to electric buses. This strategy includes predictive modeling to guide the electrification process, contributing to the city's plan to reduce greenhouse gas emissions by 80% by 2050 (CUTRIC, 2024).

The initiatives of electric transit come with the planning and operations of charging infrastructure. While the majority of the existing electric transit considers charging at depots, recent advances have focused more on opportunity charging. It involves short, frequent recharges at strategic points along a transit route, enabling electric transit to operate continuously without the need for extended downtime. Several studies have explored the feasibility, effectiveness and efficiency of opportunity charging. Li (2016) conducted qualitative analyses to compare the strengths and weakness of charging methods where opportunity charging exhibits the advantages of less land usage which is critical in urbanized areas but also has challenges of short ranges and charging uncertainties. Perumal et al. (2022) summarized the charging infrastructure and scheduling challenges and indicated the necessity to integrate infrastructure strategies with dynamic transit rescheduling to tackle the trade-offs between driving range and charging times. In parallel, researchers have analyzed the benefits and optimizations of transit charging and scheduling with opportunity charging. Li et al. (2022) designed such schemes for responsive transit services and the results indicate 11% total cost reduction compared with full charging strategies. Previous studies also developed planning and optimization of the opportunity charging considering charging costs, charging infrastructure prerequisites, revenues and so on (El-Taweel et al., 2017; Abdelwahed et al., 2020; Jefferies and Göhlich, 2020; Ji et al., 2023; Yousuf et al., 2024)

There are various challenges of opportunity charging including the installation costs, the vehicle routing and scheduling, and the uncertainty of when to charge. A few studies explored the shared charging infrastructure where electric transit and passenger electric vehicles can both access the charging facilities. Ye et al. (2022) consider shared electric charging hubs to accommodate multimodal transportation networks and found that shared hubs are more cost-effective compared to the sole charging of each single transportation model. Tian et al. (2022) proposed sharing charging station optimizations with electric transit and taxi which are capable of minimizing the total investment and operation costs.

Previous studies provided insights into characteristics and the evolving deployment of electric transit with shared charging infrastructure. Yet literature mainly focused on aggregated planning and system such as charging hubs rather than the operational perspectives of electric transit. Existing studies also lacks view of cost-benefit analyses tailored for public transit to connect with the shared charging infrastructure. With the increasing need of reliable and sustainable transit options, exploring diverse forms of charging infrastructure (e.g., new strategy on using shared public charging stations) and understanding corresponding service performance are still emerging to minimize the electrification costs and materialize the resources.

2.2. Electric transit charging scheduling

To our knowledge, many studies on optimization models for electric vehicles (EVs) have been reported. However, studies on electric buses show fewer numbers and less variety than EVs. This difference mainly came from an operational feature of electric buses. Buses travel a fixed route, and they are operated repeatedly. In the electric vehicle routing problem, many decisions should be made, such as arrival time, cargo level, and charging amount.

On the other hand, buses service the same route the whole day, and the arrival time to each station could be different from the expected time, but it will be tiny. Thus, the main decisions for optimizing the electric buses' operation could be two things: one is the schedule of buses for service trips, and the other is when buses visit charging stations. Rogge et al. (2018) studied electric buses to minimize the sum of several costs, including operational costs, energy consumption, and investment in electric buses and chargers. The proposed model decides the size and mix of several different types of electric buses. Liu et al. (2021) proposes an optimization model to minimize the total operating and customer waiting costs while determining the number of chargers and bus flows for a given network. In addition, the authors consider the seasonality affecting battery performance. Bagherinezhad et al. (2020) introduces a spatio-temporal charging optimization of electric buses considering the interdependency between power distribution and transit systems. Zhang et al. (2024) studies the collaborative optimization of charging schedules and passenger services to fully utilize the existing transit systems and minimize the cost of infrastructure upgrades. Feng et al. (2025) study an electric-bus duty scheduling and dynamic charging-scheduling problem at a depot/public charging facility, explicitly accounting for time-varying electricity tariffs and stochastic trip energy consumption. Qiu et al. (2026) proposes a robust integrated scheduling framework for multi-route electric bus operations that coordinates charging at a shared public hub station by jointly optimizing timetables, vehicle assignment, and charging schedules under stochastic traffic and battery energy-consumption uncertainties.

Similarly, many works focused on charging stations' location and charging schedules for electric buses (Hu et al., 2022; Wang et al., 2023; Rizopoulos and Gkiotsalitis, 2025). Kullman et al. (2021) propose an electric vehicle routing problem as a Markov decision process where public charging stations are considered. Most existing studies on electric bus operations concentrate on optimizing routes in conjunction with the planning or construction of dedicated charging infrastructure. However, investing in new charging facilities can be cost-prohibitive and, in some cases, unnecessary. This paper introduces an optimization model for scheduling electric buses along fixed routes while leveraging existing public charging stations as shared infrastructure. By utilizing shared charging facilities, smaller transit operators, who may lack the resources to install proprietary systems, can overcome financial barriers and more feasibly integrate electric buses into their services.

2.3. Passengers commuting pattern and behaviors on transit

In addition to the operational and planning perspectives, various studies have explored the human factors and passengers' behaviors in response to electric transit. Nguyen and Pojani (2023) surveyed university students to investigate people's intention to use electric buses and found that most participants are positive about electric transit and interested in its safety, security, and innovation. Flaris et al. (2023) explored bus users' preferences for electric buses and results indicated users perceive electric buses as reliable and safe, in line with previous research (Kwon et al., 2020). Sunitiyoso et al. (2022) investigated commuters' preferences towards electric transit where results revealed that bus and train are the most preferred modes among commuters, and electric transit is preferable compared to a conventional bus.

Previous research found electric transit are strongly supported by communities and users. In early adoption stage of transit agencies, there could be a mixture of electric and non-electric vehicles operating different lines and stations. Users' preferences on conventional or electric bus vehicles may affect their choices on transit stop and service trip selections. Yet existing findings mainly focused on qualitative analyses of users' preferences and trends rather than assessing the quantitative and systematic effects on passenger's choices and travel patterns on electric transit, especially for commuting. Several studies explores transit users' commuting

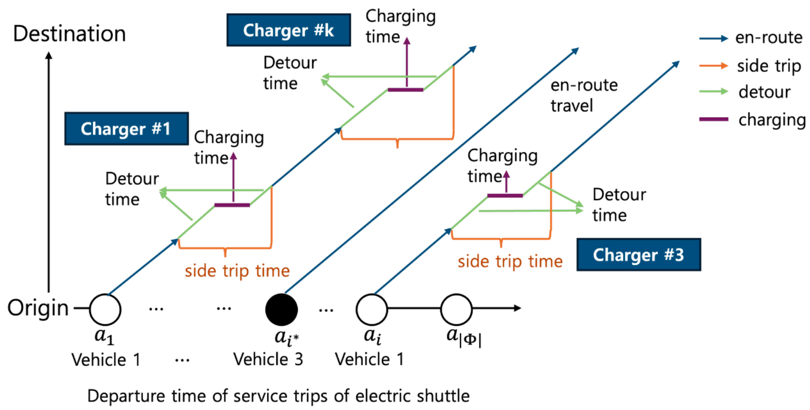


Fig. 1. Concept of electric buses with shared charging stations.

patterns and departure time choices (Tian et al., 2007; Tang et al., 2020a,b; Liang et al., 2024; Xu et al., 2024; Stokkink et al., 2025) yet they mainly focuses on the congestion, crowding management or mode choice perspectives rather than the electric transit operations. In the context of electric buses utilizing shared charging stations, as considered in this study, certain service trips may require side trips for charging, resulting in additional detours and charging times (e.g., 5–8 minutes). These operational adjustments can influence passengers' departure time choices and their selection of service trips. As demand for electric transit services grows, it becomes increasingly important to understand and model the interactions between traveler behavior and electric bus operations. To this end, this study develops a passenger user equilibrium model to evaluate the impacts of electric bus operations and charging schedules on passenger trip costs and onboarding patterns.

3. Problem setting of electric shuttle bus with shared charging stations

In this study, we consider a commuting bus or shuttle service with a fleet of electric vehicles. We focus on community shuttle services which can be an addition to a regular transit network, a feeder route, or a pilot program and can be operated by transit agencies or private entities. We consider a typical shuttle vehicle size (e.g., accommodating 15–30 people), such shuttle services are widely adopted and function across extensive urban and suburban regions. Due to limited investment and infrastructure, the electric shuttle services do not have dedicated depot charging facilities and in this case, utilize the public charging stations alongside the service route. Therefore, the electric shuttle vehicles might have a charging side trip to nearby charging stations in certain service trips for opportunity charging (e.g., 5–8 minutes) depending on the battery levels as shown in Fig. 1. The side-trip time includes the detour time to the charger, the charging time, and the return time to the service route. We consider fast charging types for all available public charging stations and thus the side-trip times will not be significantly high but may affect passengers' travel choices. We also consider a time-variant charging cost. The electricity price can vary depending on when the bus visits a public charging station. Since the electricity price changes by the hour (Board (2025)), we use bucketized time buckets, where each time bucket has an hour-long length. We assume that all shuttles start their service with a fully charged battery. This reflects the practical assumption that buses are recharged overnight at a cheaper off-peak electricity price and is consistent with industry practice.

In this work, we consider buses serve homogeneous service trips that visit the same set of stations. A service trip means a sequence of station visits. During the commuting trip, there is a set of service trips Φ connecting the origin with the destination, for instance, this can be the residential area and the work location for example for the morning commute. We consider commuting trips with a single origin and destination. Hereinafter, we define service trips with additional charging side trips as "charging trips" and service trips without charging side trips as "noncharging trips" as indicated in Fig. 1. From an operational perspective, the electric shuttle services operate a route and have a fleet of vehicles v where each vehicle has a maximum battery capacity B (in kWh). There is a total of $|\Phi|$ service trips between the origin and the destination of the given route. The scheduled departure time of the service trip i is given by a_i. Each round trip will consume energy from the battery. The transit operator will thus determine the charging schedules of the electric shuttle services where for certain service trips the shuttle will take a detour to a nearby public charging station for a short period of charging. We assume that the selected public fast-charging stations have sufficient capacity to serve shuttle buses without significant queuing delays. To reflect less favorable conditions, we also evaluate scenarios in which certain stations are unavailable, requiring shuttle buses to consider alternative charging sites. In addition, the model allows shuttle buses to opportunistically recharge during idle periods between service trips when this is operationally efficient. From the passenger perspective, each service trip refers to an electric vehicle with a full physical capacity to accommodate a total S passengers. When boarding passengers occupy all the available capacity, the service trip is at full capacity S, and some passengers may incur additional queuing time to wait for the next service run. On the demand side, a total of N homogeneous individuals need to commute using the electric bus services during the peak periods. We consider homogeneous passengers who have identical preferred departure times corresponding to a preferred departing service trip a_*. Passengers are assumed to have access to shuttle charging schedules and detour-related information in advance. This ensures that potential impacts on travel time due to charging or detours are accounted for in the passenger onboarding problem.

The following sections introduce the optimization model to devise charging schedules of the shuttle services and the integrated passenger user equilibrium model to assess the onboarding ridership in the transit system with charging schedules. Table 1 lists the parameters and variables used in the study.

4. Optimizing shuttle bus schedule with shared charging station

In the first step, we develop an optimization model to construct a charging schedule for a fleet of electric shuttles. For a set of service trips Φ, the optimization model finds the optimal charging schedule of each electric shuttle and a sequence of service trips. Service trips are categorized into charging and noncharging trips. The noncharging trip is the same as the conventional bus trip. However, the charging trip includes single or multiple charging station visits. The travel time and distance of the charging trip may be longer than the conventional bus trip because of detours for visiting charging stations as shown in Fig. 1. In a charging trip, the total travel distance increases due to detours made to access charging stations. Charging costs are time-dependent, as we incorporate a time-varying electricity pricing scheme, which reflects real-world practices adopted by many urban regions. Both charging and non-charging trips are constrained to be completed prior to the start of their subsequent service trips to ensure operational feasibility and service continuity.

This study aims to develop novel operational concept and framework with shared charging infrastructure, thereby the optimization is focusing on charging schedules and costs which are crucial to explore the operational feasibility and performance of electric buses under the shared charging settings. Our optimization model aims to minimize the charging cost while satisfying the electric shuttles' battery and time feasibility. Battery feasibility is defined as the State of Charging (SoC) level of an electric shuttle, which should be in a predefined range. Time feasibility is determined by whether all service trips ended before their next service trip so that every service trip can start without any delay in their service start time. The optimization for the charging schedule is formulated as the following mixed-integer programming model, and the notation is provided in Table 1.

Table 1
Summary of notation.

Sets	
$0, Q+1$	Initial depot and end depot
Φ	Set of service trips $\Phi := \{1, 2, \dots, Q\}$
Φ_0	$\Phi_0 := \Phi \cup \{0\}$. Similarly, $\Phi_{Q+1} := \Phi \cup \{Q+1\}$, and $\Phi_{0,Q+1} := \Phi \cup \{0, Q+1\}$
$K(i)$	Set of indices of charging location options for service trip i
T	Set of time buckets. Each bucket's length is an hour.
Variables	
x_{ij}	Binary decision variable to indicate if service trips i and j are served consecutively by the same bus not
y_i	Nonnegative continuous decision variable for the remaining battery energy level at the beginning of the service trip i (kWh)
$z_{ik\tau}$	Binary decision variable for k-th recharging position for the service trip i at time bucket τ. If a bus of trip i arrives to k-th recharging position at time bucket τ, then $z_{ikt} = 1$, and 0 otherwise.
w_{ik}	Binary decision variable for k-th recharging position for the service trip i. If a bus of trip i visits k-th recharging position, $w_{ik} = 1$ and 0 otherwise
p_{ik}	Nonnegative continuous decision variable for the recharging amount for the k-th recharging position during the service trip i (kWh)
Parameters	
v	Predefined number of buses
a_i	Departure time of service trip i (hour)
t_i	En-route travel time of the service trip i (hour)
B	Battery capacity
$\hat{B}_1, \hat{B}_2$	Lower and upper bounds for vehicle battery capacity (kWh)
σ	Minimum charging ratio
$\hat{T}$	Latest hour that the service must end (hour)
ρ_{ik}	Additional energy to visit charger k in service trip i (kW)
$\hat{\rho}_{ik}$	Required energy to travel from service trip i's route to charger k (kW)
η_{ik}	Additional time to travel from service trip i's route to charger k
$\hat{\eta}_{ik}$	Required time to visit charger k in service trip i
δ_{ik}	Recharging rate for service trip i at the k-th recharging position (h/kWh)
h_{ij}	Energy consumption for service trip j when service trip i is the preceding one (kWh)
β_{ik}	Energy consumption of the travel from the $(k-1)$-th recharging position to the k-th charging position on service trip i (kWh)
γ_{ik}	Travel time from the $(k-1)$-th recharging position to the k-th recharging position on service trip i (kWh)
$c_{ik\tau}$	Energy cost when a bus of service trip i recharges its battery on k-th position at time bucket τ
Δ	Length of the time bucket
$\bar{M}$	A sufficiently large number, used for the big M method

$$\min \sum_{i\in\Phi}\sum_{k\in K(i)}\sum_{t\in T} c_{ikt} z_{ikt} p_{ik} \tag{1a}$$

$$\text{s.t.} \sum_{i\in\Phi} x_{0i} \le v \tag{1b}$$

$$\sum_{j\in\Phi_{Q+1}, i\neq j} x_{ij} = 1 \qquad \forall i \in \Phi, \tag{1c}$$

$$\sum_{i\in\Phi_0, i\neq j} x_{ij} - \sum_{i\in\Phi_{Q+1}, i\neq j} x_{ji} = 0 \qquad \forall j \in \Phi, \tag{1d}$$

$$\sum_{t\in T} z_{ikt} = w_{ik} \qquad \forall k \in K(i), i \in \Phi, \tag{1e}$$

$$\sigma B w_{ik} \le p_{ik} \le B w_{ik} \qquad \forall k \in K(i), i \in \Phi, \tag{1f}$$

$$a_i + t_i x_{ij} + \sum_{k\in K(i)} \left(\eta_{ik} w_{ik} + \delta_{ik} p_{ik}\right) - \bar{M}(1 - x_{ij}) \le a_j \qquad \forall i \in \Phi_0, j \in \Phi_{Q+1}, i \neq j, \tag{1g}$$

$$a_i + t_i x_{i,Q+1} + \sum_{k\in K(i)} \left(\eta_{ik} w_{ik} + \delta_{ik} p_{ik}\right) - \bar{M}(1 - x_{i,Q+1}) \le \hat{T} \qquad \forall i \in \Phi, \tag{1h}$$

$$y_j \le y_i - h_{ij} x_{ij} + \sum_{k\in K(i)} \left(p_{ik} - \rho_{ik} w_{ik}\right) + B(1 - x_{ij}) \qquad \forall i \in \Phi, j \in \Phi_{Q+1}, i \neq j, \tag{1i}$$

$$y_i + \sum_{\kappa=0}^{k-1} \left(p_{i\kappa} - \beta_{i\kappa} - \rho_{i\kappa} w_{i\kappa}\right) - \beta_{ik} - \hat{\rho}_{ik} w_{ik} \ge \hat{B}_1 \qquad \forall k \in K(i), i \in \Phi, \tag{1j}$$

$$y_i + \sum_{\kappa=0}^{k-1} \left(p_{i\kappa} - \beta_{i\kappa} - \rho_{i\kappa} w_{i\kappa}\right) + p_{ik} - \beta_{ik} - \hat{\rho}_{ik} w_{ik} \le \hat{B}_2 \qquad \forall k \in K(i), i \in \Phi, \tag{1k}$$

$$y_i + \sum_{k\in K(i)} \left(p_{ik} - \beta_{ik} - \rho_{ik} w_{ik}\right) \le \hat{B}_2 \qquad \forall i \in \Phi, \tag{1l}$$

$$(\tau - 1)\Delta - \bar{M}(1 - z_{ik\tau}) \le a_i + \sum_{\kappa=0}^{k-1} \left(\gamma_{i\kappa} + \delta_{i\kappa} p_{i\kappa} + \eta_{i\kappa} w_{i\kappa}\right) + \gamma_{ik} + \hat{\eta}_{ik} w_{ik} \qquad \forall k \in K(i), i \in \Phi, \tau \in T, \tag{1m}$$

$$a_i + \sum_{\kappa=0}^{k-1} \left(\gamma_{i\kappa} + \delta_{i\kappa} p_{i\kappa} + \eta_{i\kappa} w_{\kappa}\right) + \gamma_{ik} + \hat{\eta}_{ik} w_{ik} \le \tau\Delta + \bar{M}(1 - z_{ik\tau}) \qquad \forall k \in K(i), i \in \Phi, \tau \in T, \tag{1n}$$

$$x_{ij} \in \{0, 1\} \qquad \forall i \in \Phi_0, j \in \Phi_{Q+1}, i \neq j, \tag{1o}$$

$$z_{ikt} \in \{0, 1\} \qquad \forall k \in K(i), i \in \Phi, \tau \in T, \tag{1p}$$

$$w_{ik} \in \{0, 1\} \qquad \forall k \in K(i), i \in \Phi \tag{1q}$$

$$y_i \ge 0, p_{ik} \ge 0 \qquad \forall k \in K(i), i \in \Phi. \tag{1r}$$

We note that the number of electric shuttles is a parameter v. There are five decision variables: three binary decision variables, x_{ij}, $z_{ik\tau}$, and w_{ik}, and two nonnegative decision variables, y_i and p_{ik}. Binary decision variable x_{ij} is 1 if service trip j is the following trip of service trip i, and both trips are served by the same shuttle. Binary decision variables $z_{ik\tau}$ and w_{ik} are related to the charging location of the service trip i. We refer to Fig. 2 for additional explanation. Fig. 2 depicts a service trip i that departs from a start point denoted as S and has three chargers that can be visited during service. We consider a service trip as several segments of partial routes from the start point or the charger to the next potential charger or endpoint. In Fig. 2, service trip i is divided into four segments because it has three possible charging positions. If a shuttle departs node S, the shuttle has to decide whether to visit the charger. Visiting chargers incur additional travel time and energy consumption caused by detours, which are denoted as ρ and η in Fig. 2. For instance, if the shuttle only visits charger #2 the total travel time and energy consumption are $t_i + \eta_{i2}$ and $h_{ij} + \rho_{i2}$, respectively. In this situation, the binary decision variable w_{i2} is 1. Also, if this visit happens at time bucket τ, binary decision variable $z_{i2\tau}$ is 1. In a service trip, a shuttle can visit multiple chargers, but each can be visited only once. Additionally, the shuttle must arrive at node S before the start time of the following service trip with a battery level higher than the allowed desired value.

The objective (1a) minimizes the total charging costs. Constraints (1b)–(1d) are electric buses scheduling constraints. Each service trip can be served by only one bus, and the maximum number of employed buses is bounded by v. Constraint (1e) ensures that charger $\#k$ can be visited only once for service trip i in whole buckets. Constraint (1f) bounds the charging amount, which can happen only when the bus visits charger $\#k$, ranging from the minimum charging restriction amount σB to its battery capacity B. Constraints (1g) and (1h) are time feasibility constraints. Constraint (1g) enforces that the end time of service trip i should be earlier than the start time of service trip j if trips i and j are consecutive trips on the same bus. The end time of service trip i is the sum of the start time of trip i (a_i), the en-route travel time of trip i (t_i), total detour time of trip i $\left(\sum_{k\in K(i)} \eta_{ik} w_{ik}\right)$, and total charging time of trip i $\left(\sum_{k\in K(i)} \delta_{ik} p_{ik}\right)$. Constraint (1h) enforces that the last service trip should end before the available service end time. Constraints (1i)–(1l) are battery feasibility constraints. Constraint (1i) ensures that battery level when trip j starts depends on battery level whentrip i starts, consumed

Table 2
Notations for passenger equilibrium model.

n_i	Number of onboarding passengers in service trip i
$\zeta(a_i)$	Schedule delay costs for passengers taking service trip departing at a_i
$c(a_i)$	Crowding costs for passengers taking service trip departing at a_i
$q(a_i)$	Queuing costs for passengers taking service trip departing at a_i
$TC(a_i)$	The generalized trip costs for passengers taking a service trip departing at a_i
l_i	Total in-vehicle time of service trip i, i.e., $l_i = \sum_{k\in K(i)} \eta_{ik} w_{ik} + \sum_{k\in K(i)} \delta_{ik} p_{ik} + t_i$
a_{i^*}	Passengers' preferred service trip corresponding to the preferred work starting time
ϵ	Unit costs of early arrival
φ	Unit costs of late arrival
N	Total number of passengers
S	Capacity of electric bus vehicle
$\bar{p}$	Uniform fare
υ	Equilibrium trip costs in the system

energy for trip i and detour in trip i, and charged energy in trip i. Constraint (1j) enforces that the battery level must be higher than the predefined lower bound $\hat{B}_1$. Constraints (1k) and (1l) enforce that the battery level cannot exceed the predefined upper bound $\hat{B}_2$. Constraints (1m) and (1n) stand for the time bucket in which the bus visits charger $\#k$. Decision variable $z_{ik\tau}$ becomes 1 only if the visiting time is in the range $[(\tau-1)\Delta, \tau\Delta]$.

We note that the optimization model (1) can be easily extended to consider multiple routes. All decision variables will have an additional index for routes, and all constraints also be replicated for each route. When m routes are considered, the model will be m times larger than model (1). However, the model with multiple routes can be decomposed into individual routes if the routes are mutually independent.

5. Assessment of passenger onboarding patterns with mixture of charging trips and non-charging trips

The aforementioned scheduling optimization model yields the corresponding departure times and trip trajectory of each service trip during the commuting period. As a result, some trips involve additional side trips with detours and charging times which affect passengers' choices of departure times for commuting. Passengers make their departure time choices or bus choices based on the costs associated with trips, including their scheduling time costs, crowding and queuing costs, and additional detour and charging time costs if a trip requires bus charging. This section formulates a general passenger onboarding user equilibrium model to devise the number of onboarding passengers in each trip given the optimized electric shuttle charging schedules with side trips. Table 2 shows additional notations for passenger equilibrium model.

5.1. Generalized commuting trip cost of passengers

We first characterize passengers' trip costs for taking electric bus trips. A commuter who takes a service trip departing at a_i encounters a uniform en-route time cost e, a uniform fare cost p, a schedule delay cost $\zeta(a_i)$ if he/she boards the service trips early or late other than the preferred service trip, a queuing time cost $q(a_i)$, a crowding cost $c(a_i)$, a charging side trip cost total detour time $\left(\sum_{k\in K(i)} \eta_{ik} w_{ik}\right)$, and total charging time $\left(\sum_{k\in K(i)} \delta_{ik} p_{ik}\right)$ if the service trip involves a charging side trip as described in Fig. 2.

Consequently, the generalized trip costs for passengers taking a service trip a_i are expressed by:

$$TC(a_i) = \zeta(a_i) + c(a_i) + q(a_i) + \theta \cdot \lambda \cdot \Big(\sum_{k\in K(i)} \eta_{ik} w_{ik} + \sum_{k\in K(i)} \delta_{ik} p_{ik} \Big) + \bar{p} + t_i, \tag{2}$$

where $\zeta(a_i)$ represents the schedule delay costs for charging trips and non-charging trips. The schedule costs are the value of disutility of individual passenger who cannot arrive in workplace on their preferred time. In our study, passengers incur disutilities if boarding on service trips other than the preferred service trip a_{i^*}. Based on the literature, schedule delays costs are equal to zero if individual taking the preferred service trip and thus arrives in the workplace at preferred working time. The $\zeta(a_{i^*})$ is therefore given by:

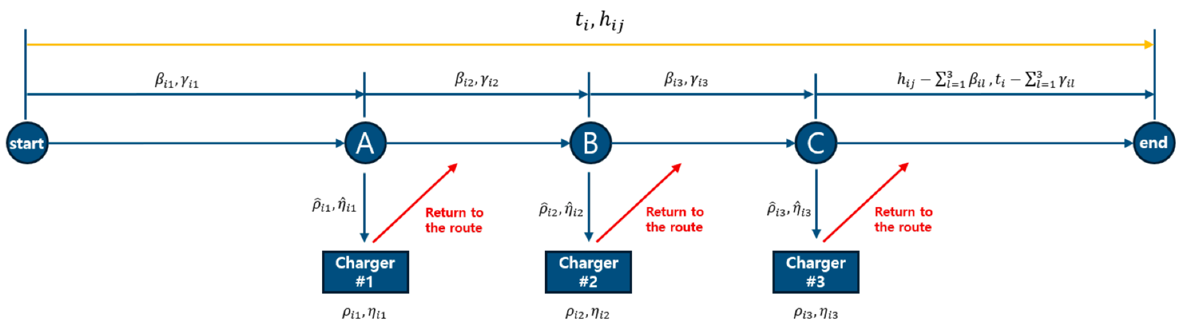


Fig. 2. Example of a service trip.

$$\zeta(a_i) = \begin{cases} \epsilon(a_* - a_i) & \text{if } a_i < a_* \\ 0 & \text{if } a_i = a_* \\ \kappa(a_i - a_*) & \text{if } a_i > a_{i*} \end{cases} \tag{3}$$

where the ϵ and φ are the unit cost of arriving early and arriving late respectively.

The crowding costs in public transit has been discussed in various studies and calculated as the number of passengers and the duration time of the trip (De Palma et al., 2017; Tang et al., 2020a; Liang et al., 2024; Xu et al., 2024). We consider the crowding costs as follows:

$$c(a_i) = g(n_i) \times (t_i + \sum_{k\in K(i)} \eta_{ik} w_{ik} + \sum_{k\in K(i)} \delta_{ik} p_{ik}), \forall i \in \Phi \tag{4}$$

in which n_i is the total number of passengers who would get onboard on charging trips or non-charging trip. In what follows, for convenience in the mathematical analysis, n_i is treated as a continuous variable for the optimization problem. $g(n_i)$ is the unit costs of charging trip or non-charging trips' crowding, which is assumed to be convex and monotonically increasing with n_i and is twice differentiable (Tian et al., 2007; De Palma et al., 2017; Tang et al., 2020a). l_i is the total in-vehicle time which is the sum of en-route time, charging time and detour times, i.e., $l_i = \sum_{k\in K(i)} \eta_{ik} w_{ik} + \sum_{k\in K(i)} \delta_{ik} p_{ik} + t_i$.

During the commuting period, passengers can get onboard in the vehicle when it is not reaching full capacity or passengers have to wait for the next upcoming trip to get onboard and thus they incur extra queuing time costs if the trip is reaching its full capacity. Due to these facts, we can infer that the equilibrium condition of charging trips and non-charging trips' queuing time costs can be indicated as follows mathematically:

$$q(a_i) = \begin{cases} 0, & \text{if } n_i < S,\ \forall i \in \Phi \\ \geq 0, & \text{if } n_i = S,\ \forall i \in \Phi \end{cases} \tag{5}$$

The fourth component of Eq. (2) is the charging side trip cost consisting of charging time costs and detour time costs where $\theta = \begin{cases} 1, & \text{if trip type is charging trip} \\ 0, & \text{if trip type is non-charging trip} \end{cases}$, and$\lambda$is the unit cost of the charging side trip time.

The last two components are uniform fare and en-route time costs which are constant as in the same bus line which has no effect on passengers' departure time choices.

In equilibrium, commuters face the trade-offs between schedule delay cost, queuing time costs, crowding costs, charging side trip costs, and the equilibrium generalized trip cost can be simplified as

$$v(a_i) = \max\{\epsilon(a_* - a_i), \varphi(a_i - a_*)\} + c(a_i) + q(a_i) + \theta \cdot \lambda \cdot \left(\sum_{k\in K(i)} \eta_{ik} w_{ik} + \sum_{k\in K(i)} \delta_{ik} p_{ik} \right) \tag{6}$$

5.2. User equilibrium for passengers with mixture of charging and noncharging trips

At equilibrium, no commuter can reduce their generalized trip costs by unilaterally changing service trips. Hence commuters taking the charging or non-charging trips incur the same values of generalized trip costs where each specific cost (i.e., crowding, charging, detouring) can be different.

As a result, if a service trip is utilized by any commuter, the generalized trip costs associated with these service run are the same (i.e., equilibrium generalized trip costs), and if a service trip is empty (i.e., not utilized by any commuter), then the generalized trip cost associated with such service trips are no less than the equilibrium generalized trip cost.

The equilibrium solution is the number of onboard passengers in each service trip. The equilibrium condition can be mathematically expressed as

$$\begin{cases} v(a_i) = v, & \text{if } n_i > 0,\ \forall i \in \Phi \\ v(a_i) \geq v, & \text{if } n_i = 0,\ \forall i \in \Phi \end{cases} \tag{7}$$

where v is the equilibrium generalized trip cost, and $v(a_i)$ is the generalized trip cost of taking charging or noncharging service trip departing at a_i with number of n_i onboard passengers.

Based on the above condition and the generalized trip costs formulations, the equilibrium can be derived by solving the number of onboard passengers in each service trip with the following optimal minimizing problem:

$$\min \quad U(\mathbf{n}) = \sum_{i\in\Phi} G(n_i) \cdot \left(t_i + \sum_{k\in K(i)} \eta_{ik} w_{ik} + \sum_{k\in K(i)} \delta_{ik} p_{ik} \right) + \sum_{i\in\Phi} n_i \cdot \zeta(t_i) + \sum_{i\in\Phi} n_i \cdot \theta \cdot \lambda \cdot \left(\sum_{k\in K(i)} \eta_{ik} w_{ik} + \sum_{k\in K(i)} \delta_{ik} p_{ik} \right) \tag{8a}$$

$$\text{s.t.} \quad \sum_{i\in\Phi} n_i = N \tag{8b}$$

$$n_i \leq S, \qquad \forall i \in \Phi \tag{8c}$$

$$n_i \geq 0, \qquad \forall i \in \Phi \tag{8d}$$

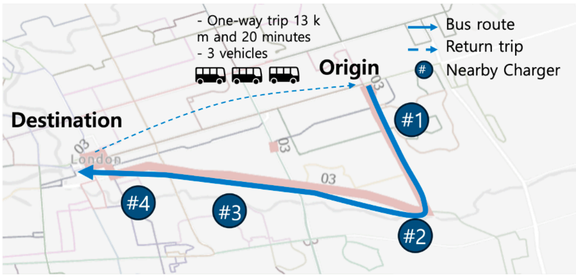


Fig. 3. Operation of electric shuttle bus.

The above objective function has no economic interpretation in line with Beckman's mathematical model and user equilibrium (Beckmann et al. (1956), Tang et al. (2020a)). It is constructed where the solution of the optimization model (i.e., the number of onboarding passengers in each service trip) is equivalent to the aforementioned user equilibrium. The equivalency proof of the equilibrium condition is provided in Appendix 1. As indicated in the appendix, passenger equilibrium for a given charging schedule is unique.

The first part of the objective function, $G(n_i) = \int_0^{n_i} g(x)dx$ is the integral of unit crowding costs function. The second part of the objective function is the aggregated scheduling delay costs for each trip. The third part of the objective function is composed of the aggregated charging and detour costs for charging trips. Constraint (8b) indicates all passengers be served and constraint (8c) indicates the rigid physical capacity.

The optimization problem is convex, the equilibrium solution of the number of onboard passengers can be obtained by solving the following first-order condition:

$$\begin{cases} n_i\{\zeta(a_i) + c(a_i) + q(a_i) + \theta \cdot \lambda \cdot (\sum_{k\in K(i)} \eta_{ik} w_{ik} + \sum_{k\in K(i)} \delta_{ik} p_{ik}) - \upsilon\} = 0, \quad \forall i \in \Phi \\ \zeta(a_i) + c(a_i) + q(a_i) + \theta \cdot \lambda \cdot (\sum_{k\in K(i)} \eta_{ik} w_{ik} + \sum_{k\in K(i)} \delta_{ik} p_{ik}) \ - \upsilon \geq 0, \quad \forall i \in \Phi \\ q(a_i)(S - n_i) = 0, \quad \forall i \in \Phi \\ q(a_i) \geq 0, \quad \forall i \in \Phi \\ n_i \leq S, \quad \forall i \in \Phi \\ n_i \geq 0, \quad \forall i \in \Phi \\ \sum_{i\in\Phi} n_i = N \\ \upsilon \geq 0 \end{cases} \tag{9a}$$

The first two equations indicate that if there are passengers taking certain service trips (i.e., $n_i \geq 0$), the generalized trip costs of these service trips are the same. If the generalized trip costs of service trips are greater than the equilibrium costs υ, no passengers take such service trips (i.e., $n_i \leq 0$). The third and fourth equations indicate if the number of onboard passengers is smaller than the full capacity, the queuing cost is zero (i.e., $q(a_i) = 0$). If service trips are at full capacity, the queue may develop (i.e., $q(a_i) \geq 0$). The last three equations are the original capacity, demand, and definitional constraints.

6. Case study and computational results

The above sections devise the optimal charging schedules for electric shuttle buses using shared public charging stations, and the corresponding user equilibrium patterns. In this section, we analyze a numerical example to investigate the performance of electric shuttle buses and passengers' commuting choices with charging schedules.

6.1. Operational setting and parameters

We consider a commuting shuttle service in City of London, Ontario connecting the residential area to the downtown via a transit route as shown in the Fig. 3 below. The route trajectory is used by various bus lines in the city. The parameters and characteristics of the case study is listed as follows:

Shuttle bus vehicle configuration and trips

The shuttle bus headway is set as 15 minutes with trip distance 13 km and 20 minutes trip time. The solid line in Fig. 3 represents the trip route which is a typical bus commuting trajectory in the City of London, Ontario. The first bus trip of the day departs at 6:00 am, and the total fleet is set as three bus vehicles operating the trips. The capacity of the shuttle bus is 20 persons referring to typical commuting shuttle bus (e.g., campus shuttle, school bus). Without loss of generality, we consider the morning operations and charging stations are located at one direction of the route as shown in Table 3 and Fig. 3:

Electric bus battery and electricity price

Table 3
Shuttle bus timetable.

Trip Index i	Origin (depart)	Destination	Origin (return)
1	6:00 am	6:20 am	6:40 am
2	6:15 am	6:35 am	6:55 am
…	…	…	…
19	10:45 am	11:05 pm	11:25 pm
20	11:00 am	11:20 am	11:40 am

Table 4
Charging regimes description.

Charging Regime	Battery lower bound	Battery upper bound	Time-of-Use Scale Factor
I - New Battery and uniform electricity price	0.1	0.9	1.00
II - Battery Degradation Reduction and uniform electricity price	0.2	0.8	1.00
III - New Battery and time-varying price	0.1	0.9	6pm - 8am: 0.87 8am - 6pm: 1.22
IV - Battery Degradation Reduction and time-varying price	0.2	0.8	6pm - 8am: 0.87 8am - 6pm: 1.22

Table 5
Costs for each charging location.

Charging location	Detour Time (mins)	Cost ($/kWh)
Location #1	1	0.31
Location #2	3	0.25
Location #3	1	0.29
Location #4	1	0.31

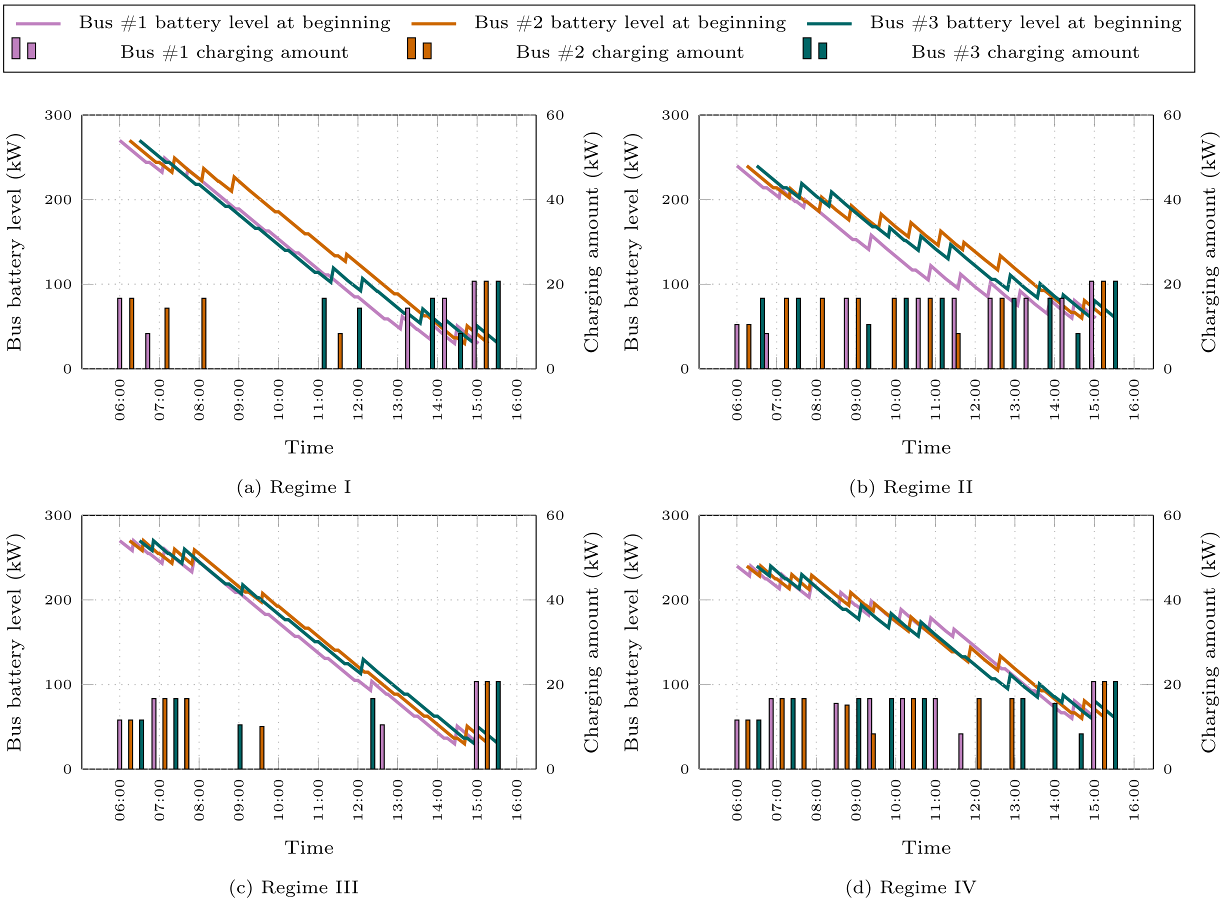


Fig. 4. Battery level and charging amount - minimum charging ratio $\sigma = 1\%$.

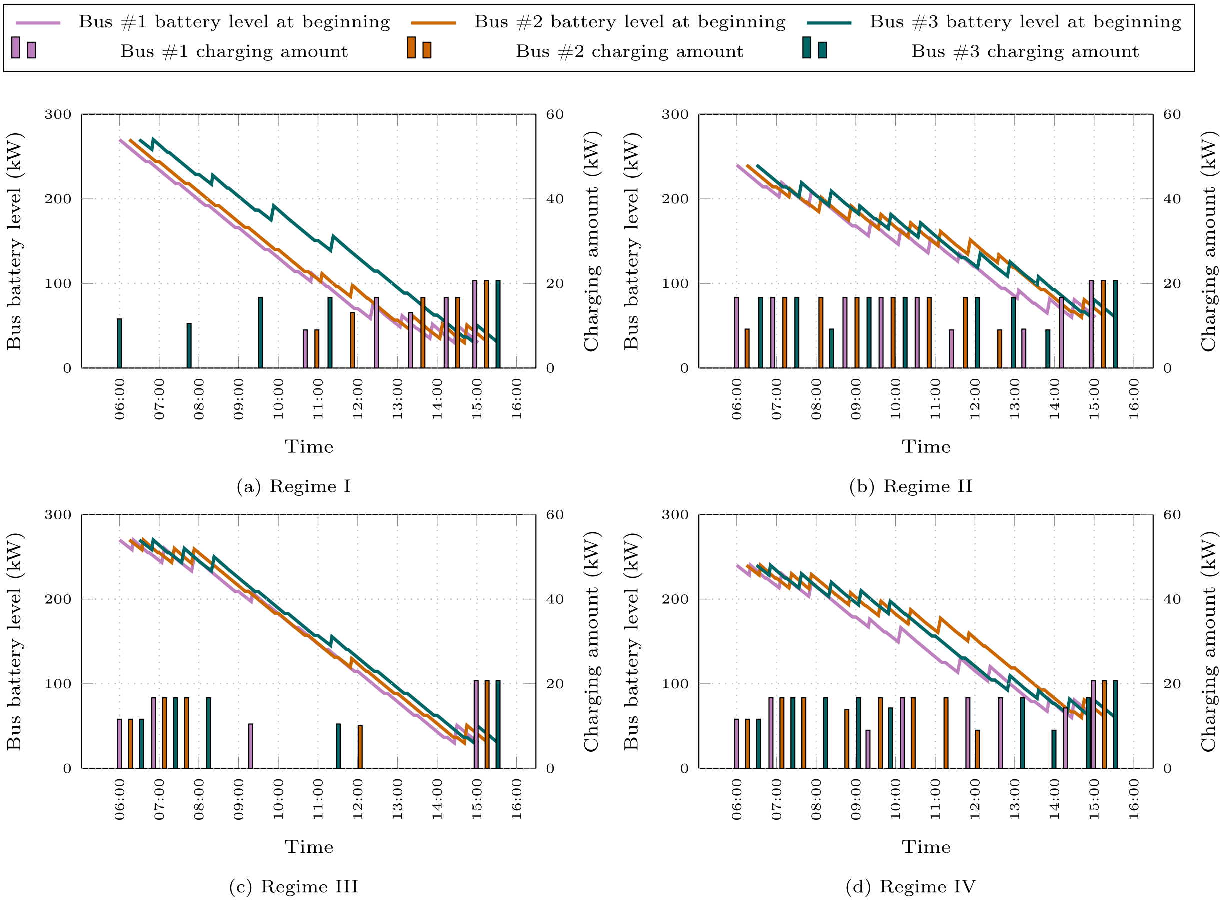


Fig. 5. Battery level and charging amount - minimum charging ratio $\sigma = 3\%$.

Based on prior studies such as Schneider et al. (2014), the battery capacity and energy consumption ratio of the shuttle vehicle is set as 1 kW per kilometer. Each vehicle is equipped with a 300 kWh battery, with its state of charge constrained between a specified lower and upper bound. The shuttle bus initiates charging when the battery level reaches the lower bound and terminates charging upon reaching the upper bound of its capacity. We also consider time-variant electricity costs. The time-variant charging cost is set to the charging cost of each charger multiplied by a scaling factor depending on the time-of-use (Board, 2025).

Based on the above, four regimes are considered as summarized in Table 4. The first one represents the shuttles are equipped with new batteries where the battery can be charged to 90% full capacity, the second regime represents a case where the shuttles have old batteries and thus a charging scheme that limits the battery charge to preserve battery health condition and reduce degradation is needed. In this case, the battery is charged to 80% of the maximum capacity. The third and fourth schemes represent new battery conditions and battery degradation reduction with time-variant electricity prices.

Charging Stations Data and Time-of-Use Electricity Prices

In this case study, we consider four fast charging stations alongside the shuttle route. The charging ratio is 4.176 kW per minute which is considered a fast charging station (Agency, 2023). The charging costs of each charging location are reported in Table 5 with due consideration of the local charging rate (Chargepoint, 2025). Each charging station also involves additional detour time and charging time as shown in Table 5.

6.2. Optimal shuttle bus charging schedules

In this section, we solve the optimization model for the shuttle bus charging schedule presented in (1) using the parameters presented in Section 6.1 We note that the optimization model contains bilinear terms in the objective function of the form $c_{ikt} z_{ikt} p_{ik}$. These terms can be easily linearized by using the reformulation linearization technique (RLT) (Adams and Sherali (1990)). Also, Gurobi, a commercial solver, can solve the original model within a predefined optimality gap. Since the RLT increases the number of constraints and variables, we solve the optimization model (1) using Gurobi version 11.0.3, with all Gurobi parameters set to their default values. We note that the average computational time to solve the optimization model (1) is less than a second. The

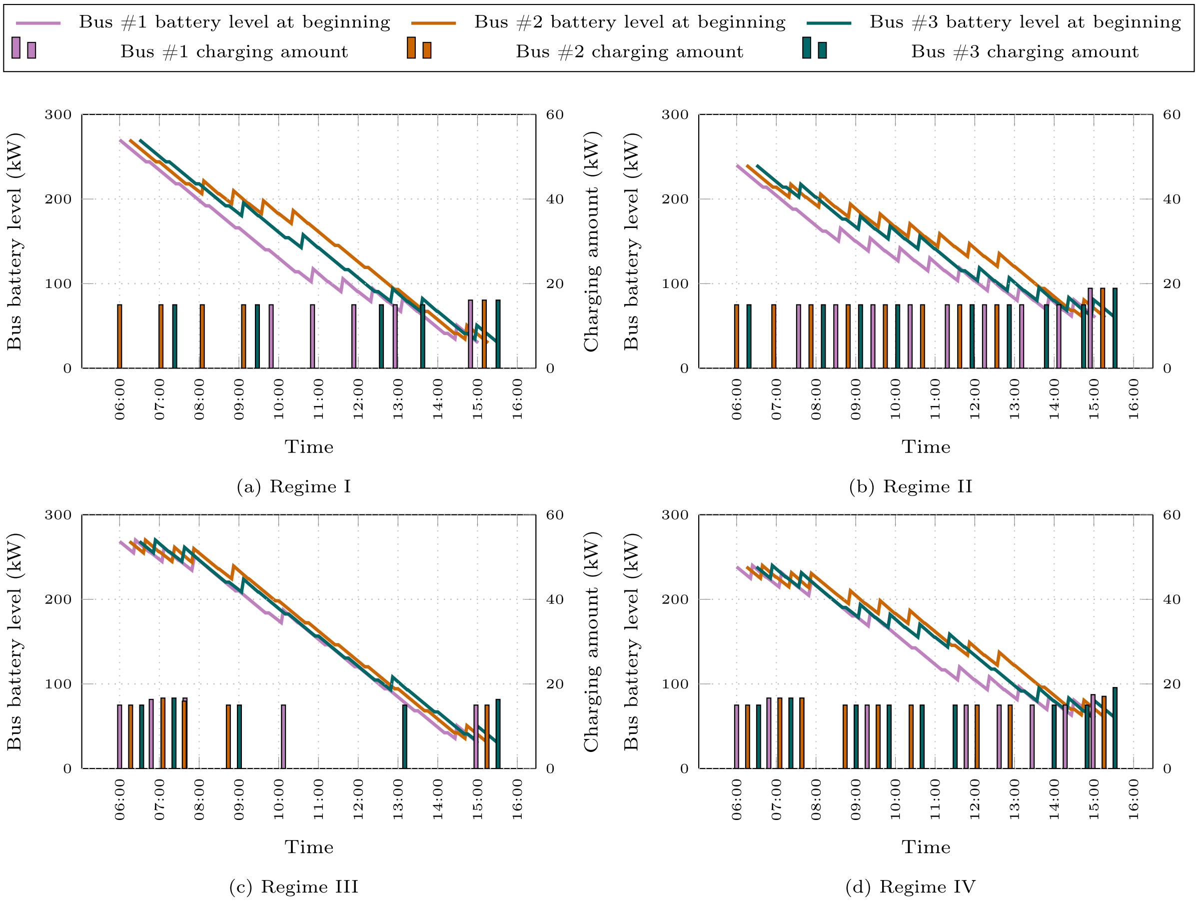


Fig. 6. Battery level and charging amount - minimum charging ratio $\sigma = 5\%$.

computational time tends to increase with the number of routes, but Gurobi can find an optimal or near-optimal solution within a reasonable time.

Figs. 4–6 illustrate the change of battery state of charge (SoC) for three electric shuttles and the associated recharging amounts when the minimum charging amount is restricted to 1%, 3%, and 5% of the battery capacity B, respectively and each subfigure shows the results for each regime. Across all figures, the results of Regime I and II (time-invariant pricing) show that electric shuttles tend to charge without time restrictions. This frequent periodic pattern in charging is particularly noticeable in Regimes II and IV with battery degradation and limitations in the charging upper and lower bounds. In Regime I, with a new battery, the electric shuttles visit charging stations less frequently than in Regime II, thereby minimizing delays caused by detours for charging station visits: in Regime I each shuttle charges five times, versus nine times in Regime II. The results of Regimes III and IV reveal an additional trend compared to those of Regimes I and II. Since the charging cost varies based on the time of day, there is a tendency to avoid charging station visits between 8:00 AM and 6:00 PM, when the price is higher. Examining the results of Regime III with a new battery, it is evident that most charging occurs before 8:00 AM. Regime IV with an old battery also results in charging strategies influenced by the time-variable price. Unlike in Regime II, where each charging station visit involved charging for the maximum available spare time, we can observe that the amount charged between 8:00 AM and 10:00 AM decreases. This is likely due to the increase in electricity prices starting at 8:00 AM. The number of station visits mirrors the flat-price cases: five per shuttle in Regime III and nine in Regime IV, suggesting that battery degradation renders the schedule less responsive to time-varying prices.

The results also indicate that imposing a minimum charging amount reduces the impact of time-varying prices (see, e.g., Figs. 4(d) and 6(d)). A noteworthy observation is that the number of charging-station visits remains unchanged even as the minimum charging amount increases. Across all figures, each shuttle visits a station five times in Regimes I and III, and nine times in Regimes II and IV. Because shuttles must serve trips at fixed departure times, the available time for charging is limited. For instance, the maximum allowable charge between consecutive service trips is only 5.56% of the battery capacity B. Consequently, the minimum-charge constraint effectively forces the schedule to prioritize battery feasibility, diminishing responsiveness to other considerations such as time - varying price. Detailed numerical results are provided in Tables B.9–B.12 in Appendix 2. We report only shuttle bus #1 in those tables, as all shuttles exhibit similar behavior.

6.3. Passengers' onboarding patterns with and without charging trips

The optimized scheduling timetables yield additional side trip time and charging time of charging service runs. Passengers who take charging service runs will incur these additional time costs compared with non-charging service runs. Therefore, passengers will make their departure time choices based on the generalized trip costs, and user equilibrium will be achieved where no commuters can reduce their generalized trip costs by unilaterally changing service trips as explained in Section 5.2. In this section, we turn to passenger onboarding computation as explained in Section 5 to explore the effects of charging schedules on passengers' behavior and electric shuttle onboarding patterns. The socioeconomic characteristics of the transit system are listed in Table 6.

The unit cost of the early arrival time and late arrival time are given with references of the city's average salaries and prior studies (Liang et al., 2024; Wardman, 2012; Secretariat, 2022). The schedule delay cost function is set as in Eq. (3). The crowding cost function is set as $g(n_i) = 40 \times n_i/s$ which is monotonically increasing with the number of in-vehicle passengers. The unit cost of the charging side trip is set as $50 \cdot (\sum_{k\in K(i)} \eta_{ik} w_{ik} + \sum_{k\in K(i)} \delta_{ik} p_{ik})$ since the side trip time is the total loss for passengers. The corresponding equilibrium passenger's onboarding patterns with different charging schedules (i.e., charging availability, new battery, battery degradation, fixed electricity price, time-variant electricity price) are shown in Fig. 7.

On the one hand, the electric shuttle vehicles leverage shared charging stations can reduce substantial investment and facility installation costs. On the other hand, the additional charging-related activities can result in different patterns of passengers' costs and peak times (full capacity) as shown in Fig. 7 and Table 7. From individual passenger's perspective, under both fixed and time-variant electricity price, the electric transit system with new battery conditions yield lower passenger's individual equilibrium trip costs. From systematic effects, passengers' trip costs vary with the distribution of the charging trips. For instance, considering minimum

Table 6
Parameters of socioeconomic characteristics.

Parameters	Value
Preferred service trip	8:30 am
Total number of passengers	500
Unit cost of travel time:	\$5.10 CAD/h
Unit cost of early arrival ϵ:	\$3.77 CAD/h
Unit cost of late arrival φ:	\$6.88 CAD/h

Table 7
Costs of different charging regimes.

Price type	Regime	$\sigma = 1\%$ - Trip costs (\$)				
		Queuing	Crowding	Schedule delay	Additional Charging-related	Individual Equilibrium
Fixed price	I	800.25	5928.71	2814.30	496.61	20.08
	II	317.00	6245.04	2955.61	1385.06	21.82
Percentage change (%)		-60.39	5.34	5.02	178.90	8.66
Time-variant price	III	887.24	5910.05	2769.64	394.63	19.92
	IV	669.06	6224.21	2887.67	1329.91	22.26
Percentage change (%)		-24.59	5.32	4.26	237.00	11.71
Price type	Regime	$\sigma = 3\%$ - Trip costs (\$)				
		Queuing	Crowding	Schedule delay	Additional Charging-related	Individual Equilibrium
Fixed price	I	438.19	5869.82	2870.81	519.31	19.40
	II	341.91	6143.40	3005.43	1321.20	21.63
Percentage change (%)		-21.97	4.66	4.69	154.42	11.50
Time-variant price	III	805.06	5987.68	2716.83	382.13	19.78
	IV	822.74	6191.08	2843.99	1230.98	22.21
Percentage change (%)		2.20	3.40	4.68	222.14	12.26
Price type	Regime	$\sigma = 5\%$ - Trip costs (\$)				
		Queuing	Crowding	Schedule delay	Additional Charging-related	Individual Equilibrium
Time-variant price	III	554.19	5708.46	2997.15	616.03	19.76
	IV	176.38	6177.32	3006.01	1353.73	21.44
Percentage change (%)		-68.17	8.21	0.30	119.75	8.47
Time-variant price	III	1124.60	5876.78	2782.16	421.73	20.41
	IV	712.84	6239.37	2848.25	1266.46	22.13
Percentage change (%)		-36.61	6.17	2.38	200.30	8.44

Table 8
Summary of results of the shuttle bus charging schedule with the availability of chargers.

Regime	Availability	σ	# visits	charge amount	charge per visit	detour dist.	avg detour dist.	detour time	avg detour time
I	50%	1%	15.34 ± 0.68	228.69 ± 1.50	14.93 ± 0.55	12.69 ± 1.50	0.83 ± 0.08	22.33 ± 4.39	1.46 ± 0.25
		3%	15.00 ± 0.12	227.18 ± 0.66	15.14 ± 0.12	11.18 ± 0.66	0.75 ± 0.04	18.07 ± 1.69	1.21 ± 0.11
		5%	15.01 ± 0.21	227.07 ± 0.67	15.13 ± 0.20	11.02 ± 0.73	0.74 ± 0.05	17.98 ± 1.58	1.20 ± 0.10
	60%	1%	15.04 ± 0.23	228.71 ± 1.03	15.21 ± 0.21	12.71 ± 1.04	0.84 ± 0.07	22.39 ± 3.37	1.49 ± 0.22
		3%	15.02 ± 0.17	227.52 ± 0.53	15.15 ± 0.16	11.52 ± 0.53	0.77 ± 0.04	18.61 ± 1.64	1.24 ± 0.11
		5%	15.01 ± 0.12	227.48 ± 0.59	15.16 ± 0.11	11.46 ± 0.61	0.76 ± 0.04	18.58 ± 1.77	1.24 ± 0.12
	70%	1%	15.01 ± 0.10	229.01 ± 0.93	15.26 ± 0.10	13.01 ± 0.93	0.87 ± 0.06	23.35 ± 3.06	1.56 ± 0.20
		3%	15.00 ± 0.00	227.77 ± 0.46	15.18 ± 0.03	11.77 ± 0.46	0.78 ± 0.03	19.25 ± 1.53	1.28 ± 0.10
		5%	15.00 ± 0.00	227.76 ± 0.48	15.18 ± 0.03	11.76 ± 0.48	0.78 ± 0.03	19.25 ± 1.60	1.28 ± 0.11
	80%	1%	15.00 ± 0.00	229.39 ± 0.84	15.29 ± 0.06	13.39 ± 0.84	0.89 ± 0.06	24.63 ± 2.80	1.64 ± 0.19
		3%	15.05 ± 0.24	227.94 ± 0.41	15.14 ± 0.23	11.93 ± 0.41	0.80 ± 0.03	19.78 ± 1.34	1.32 ± 0.09
		5%	15.00 ± 0.00	227.97 ± 0.40	15.20 ± 0.03	11.97 ± 0.40	0.80 ± 0.03	19.92 ± 1.32	1.33 ± 0.09
	90%	1%	15.00 ± 0.00	229.76 ± 0.59	15.32 ± 0.04	13.76 ± 0.59	0.92 ± 0.04	25.86 ± 1.96	1.72 ± 0.13
		3%	15.05 ± 0.27	228.14 ± 0.29	15.16 ± 0.25	12.13 ± 0.29	0.81 ± 0.02	20.44 ± 0.96	1.36 ± 0.06
		5%	15.05 ± 0.22	228.13 ± 0.31	15.16 ± 0.21	12.13 ± 0.31	0.81 ± 0.02	20.43 ± 1.02	1.36 ± 0.07
II	50%	1%	–	–	–	–	–	–	–
		3%	–	–	–	–	–	–	–
		5%	–	–	–	–	–	–	–
	60%	1%	27.76 ± 1.04	418.24 ± 1.72	15.09 ± 0.50	22.23 ± 1.72	0.80 ± 0.04	40.11 ± 4.60	1.44 ± 0.12
		3%	–	–	–	–	–	–	–
		5%	–	–	–	–	–	–	–
	70%	1%	27.38 ± 0.85	418.28 ± 1.46	15.29 ± 0.41	22.28 ± 1.46	0.81 ± 0.03	38.90 ± 4.04	1.42 ± 0.11
		3%	26.43 ± 0.67	414.72 ± 1.21	15.70 ± 0.37	18.71 ± 1.21	0.71 ± 0.03	30.59 ± 1.97	1.16 ± 0.06
		5%	25.60 ± 0.87	413.58 ± 1.55	16.17 ± 0.50	17.57 ± 1.55	0.69 ± 0.04	29.79 ± 2.09	1.16 ± 0.06
	80%	1%	27.08 ± 0.42	418.18 ± 0.78	15.44 ± 0.20	22.18 ± 0.78	0.82 ± 0.02	38.12 ± 2.17	1.41 ± 0.06
		3%	26.66 ± 0.67	415.63 ± 1.05	15.60 ± 0.37	19.63 ± 1.05	0.74 ± 0.03	31.39 ± 1.90	1.18 ± 0.05
		5%	26.26 ± 0.77	415.05 ± 1.38	15.82 ± 0.42	19.05 ± 1.38	0.72 ± 0.03	31.05 ± 2.01	1.18 ± 0.05
	90%	1%	27.00 ± 0.04	418.33 ± 0.32	15.49 ± 0.03	22.32 ± 0.32	0.83 ± 0.01	38.43 ± 1.02	1.42 ± 0.04
		3%	26.86 ± 0.63	416.17 ± 0.84	15.50 ± 0.34	20.16 ± 0.84	0.75 ± 0.02	32.07 ± 1.51	1.20 ± 0.04
		5%	26.70 ± 0.51	416.10 ± 0.89	15.59 ± 0.27	20.10 ± 0.90	0.75 ± 0.02	32.10 ± 1.52	1.20 ± 0.04
III	50%	1%	15.78 ± 0.82	228.79 ± 1.60	14.53 ± 0.65	12.72 ± 1.58	0.80 ± 0.07	22.34 ± 3.98	1.41 ± 0.20
		3%	15.02 ± 0.23	227.10 ± 0.80	15.12 ± 0.21	11.09 ± 0.80	0.74 ± 0.05	17.92 ± 1.71	1.20 ± 0.11
		5%	14.95 ± 0.22	232.13 ± 2.18	15.53 ± 0.23	11.23 ± 0.85	0.75 ± 0.05	17.93 ± 1.68	1.20 ± 0.11
	60%	1%	15.50 ± 0.64	228.37 ± 1.43	14.75 ± 0.52	12.35 ± 1.43	0.80 ± 0.07	21.55 ± 3.57	1.39 ± 0.19
		3%	15.02 ± 0.22	227.26 ± 0.68	15.13 ± 0.19	11.25 ± 0.68	0.75 ± 0.05	18.60 ± 1.66	1.24 ± 0.11
		5%	15.00 ± 0.24	232.95 ± 1.94	15.54 ± 0.25	11.53 ± 0.78	0.77 ± 0.05	18.47 ± 1.59	1.23 ± 0.10
	70%	1%	15.38 ± 0.57	228.26 ± 1.31	14.86 ± 0.47	12.24 ± 1.31	0.80 ± 0.06	21.51 ± 3.17	1.40 ± 0.17
		3%	15.03 ± 0.23	227.39 ± 0.66	15.13 ± 0.21	11.39 ± 0.66	0.76 ± 0.04	19.16 ± 1.55	1.28 ± 0.10
		5%	14.99 ± 0.10	233.36 ± 1.73	15.57 ± 0.14	11.88 ± 0.71	0.79 ± 0.05	19.31 ± 1.58	1.29 ± 0.10
	80%	1%	15.24 ± 0.47	227.97 ± 1.11	14.97 ± 0.39	11.97 ± 1.11	0.79 ± 0.06	21.05 ± 2.55	1.38 ± 0.14
		3%	15.13 ± 0.41	227.55 ± 0.58	15.05 ± 0.37	11.54 ± 0.58	0.77 ± 0.04	19.88 ± 1.35	1.33 ± 0.09
		5%	15.05 ± 0.31	233.66 ± 1.31	15.53 ± 0.30	12.17 ± 0.60	0.81 ± 0.04	19.89 ± 1.38	1.33 ± 0.09
	90%	1%	15.18 ± 0.49	227.95 ± 0.81	15.03 ± 0.40	11.95 ± 0.81	0.79 ± 0.04	21.01 ± 1.84	1.39 ± 0.10
		3%	15.12 ± 0.34	227.71 ± 0.42	15.07 ± 0.32	11.71 ± 0.42	0.78 ± 0.03	20.41 ± 1.00	1.36 ± 0.07
		5%	15.10 ± 0.34	233.64 ± 0.89	15.48 ± 0.32	12.42 ± 0.42	0.83 ± 0.03	20.41 ± 1.00	1.36 ± 0.07
IV	50%	1%	–	–	–	–	–	–	–
		3%	–	–	–	–	–	–	–
		5%	–	–	–	–	–	–	–
	60%	1%	27.94 ± 1.04	418.16 ± 1.95	14.99 ± 0.49	22.13 ± 1.95	0.79 ± 0.05	39.24 ± 5.15	1.40 ± 0.15
		3%	–	–	–	–	–	–	–
		5%	–	–	–	–	–	–	–
	70%	1%	27.43 ± 0.74	417.84 ± 1.52	15.24 ± 0.36	21.82 ± 1.53	0.80 ± 0.04	37.90 ± 4.01	1.38 ± 0.12
		3%	26.96 ± 0.27	415.27 ± 0.97	15.41 ± 0.15	19.26 ± 0.97	0.72 ± 0.03	31.20 ± 1.59	1.16 ± 0.06
		5%	26.73 ± 0.54	420.99 ± 2.28	15.76 ± 0.27	19.44 ± 1.16	0.73 ± 0.03	30.85 ± 1.76	1.15 ± 0.06
	80%	1%	27.14 ± 0.41	417.62 ± 1.04	15.39 ± 0.20	21.61 ± 1.04	0.80 ± 0.03	37.23 ± 2.89	1.37 ± 0.10
		3%	27.06 ± 0.39	415.79 ± 0.68	15.37 ± 0.21	19.78 ± 0.68	0.73 ± 0.02	31.73 ± 1.36	1.18 ± 0.05
		5%	26.92 ± 0.32	421.82 ± 1.57	15.67 ± 0.16	20.26 ± 0.82	0.75 ± 0.03	31.70 ± 1.38	1.18 ± 0.05
	90%	1%	27.06 ± 0.29	417.76 ± 0.70	15.44 ± 0.16	21.76 ± 0.70	0.81 ± 0.03	37.88 ± 1.99	1.40 ± 0.07
		3%	27.03 ± 0.16	416.12 ± 0.43	15.40 ± 0.09	20.12 ± 0.43	0.75 ± 0.02	32.41 ± 1.04	1.20 ± 0.04
		5%	27.00 ± 0.18	422.08 ± 1.05	15.63 ± 0.10	20.84 ± 0.49	0.77 ± 0.02	32.44 ± 0.99	1.20 ± 0.04

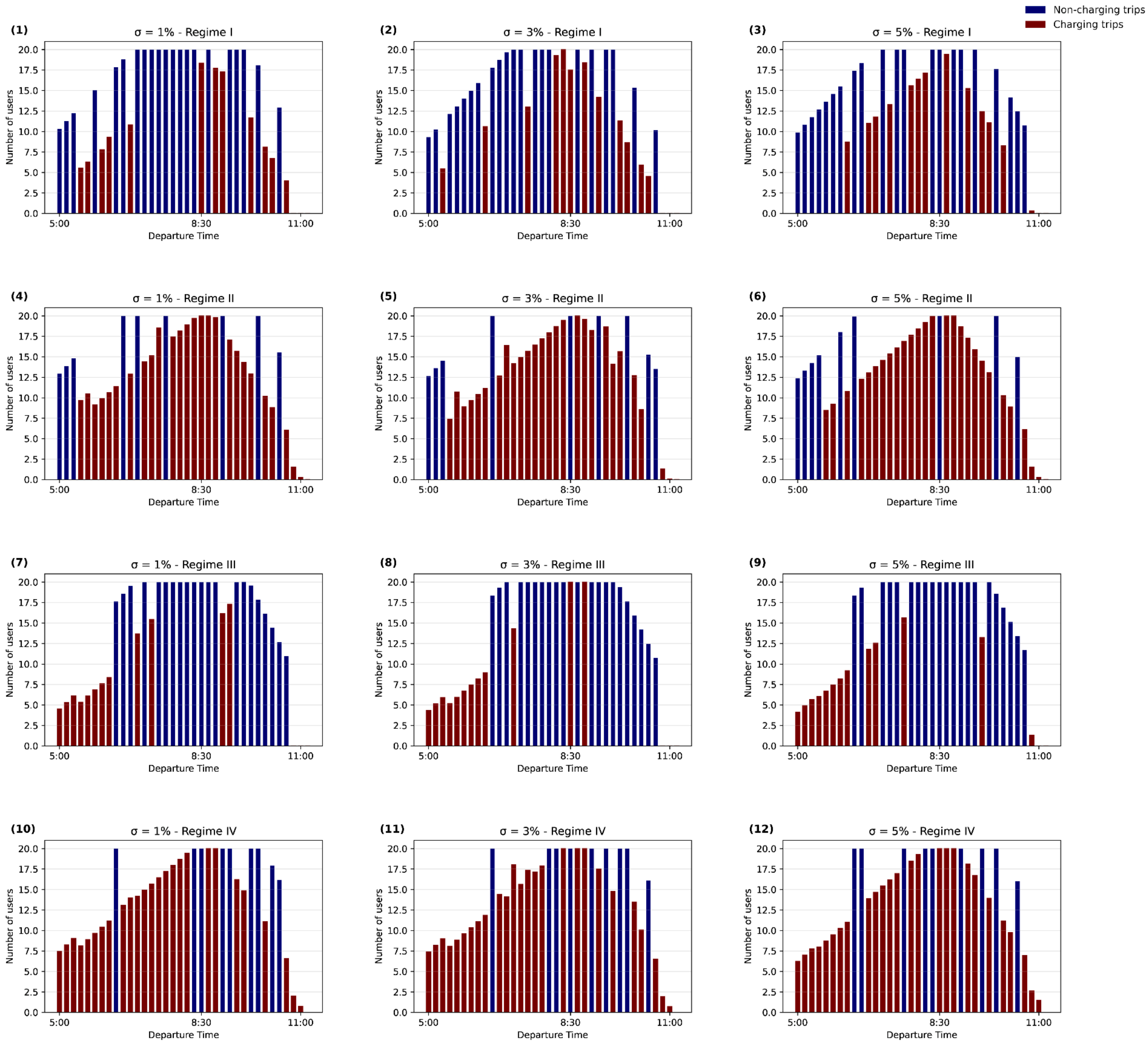
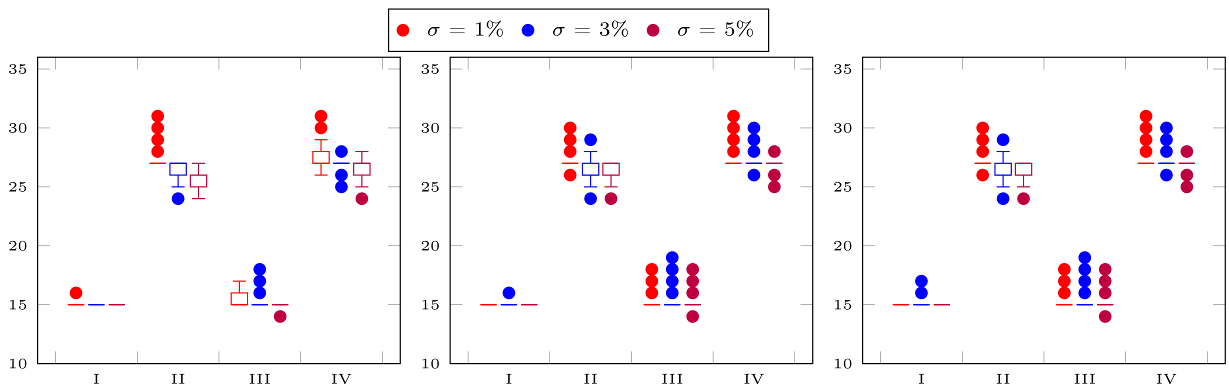


Fig. 7. Passengers' onboarding patterns with electric shuttle charging regimes.

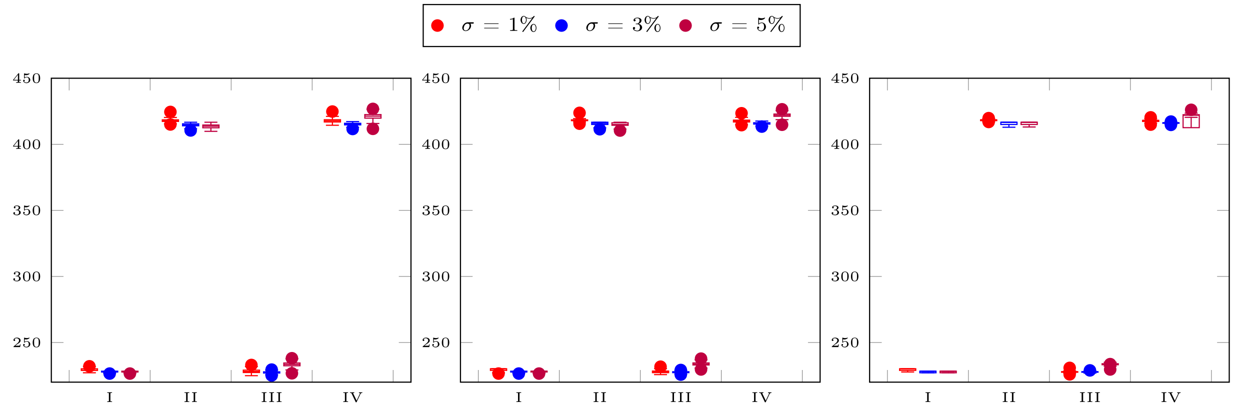


(a) Availability 70%

(b) Availability 80%

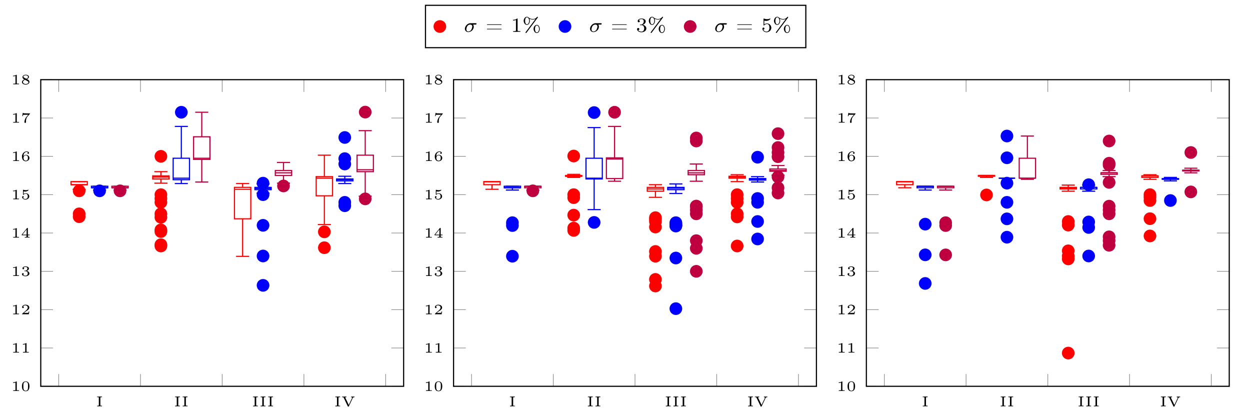
(c) Availability 90%

Fig. 8. Number of charging station visits.



(a) Availability 70% (b) Availability 80% (c) Availability 90%

Fig. 9. Total charging amount (kilowatt).



(a) Availability 70% (b) Availability 80% (c) Availability 90%

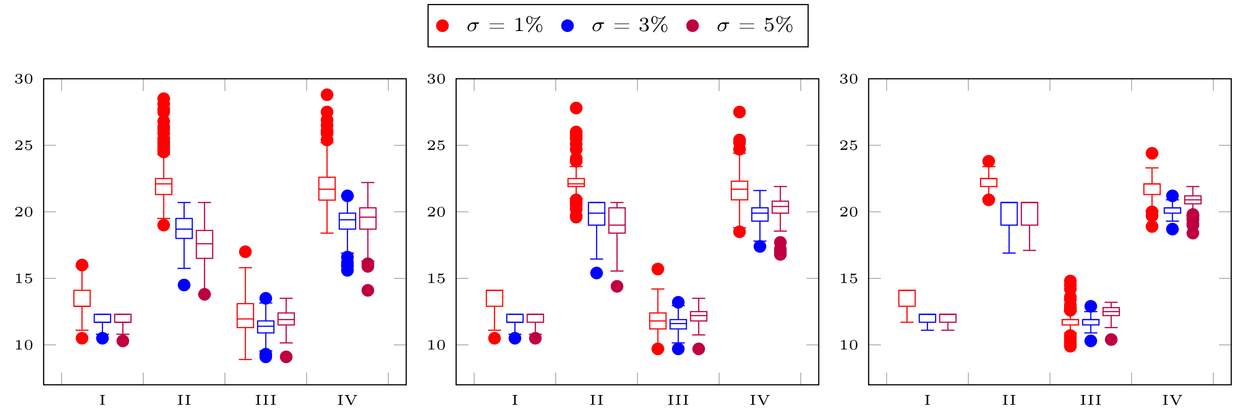
Fig. 10. Charging amount per charging station visit (kilowatt/visit).

charging amount 3%, Regime II (less battery capacity) with fixed price yield lower queuing costs, higher crowding costs, schedule delay costs and additional-charging costs compared with Regime I. Regime IV with time-varying price yield all higher trips costs than Regime III. These reflect the performance and effects of electric shuttle operations on passengers' costs and behaviors. For multiple charging scheduling solutions that yield similar objective values, operators can refer to the passengers' individual equilibrium costs, and total trip equilibrium costs (e.g., queuing, crowding, schedule delay) to further select applicable charging schedule plans.

Results also show that charging schedules affect the peak times and durations where different regimes exhibit different patterns in the distributions of full-capacity service trips and the duration of the peak times. This indicates appropriate charging schedule involvement and designs during peak period can contribute to peak congestion and demand management.

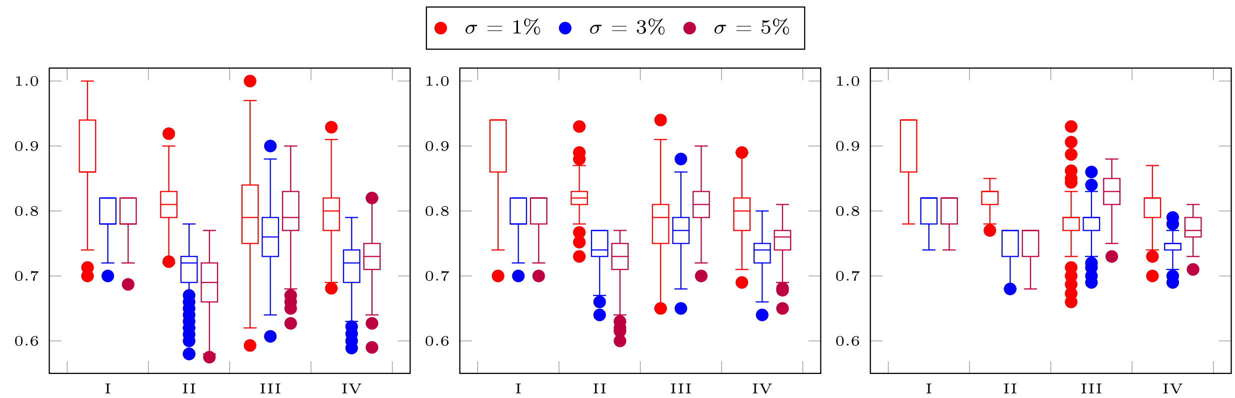
6.4. *The shuttle bus charging schedules with chargers' availability*

The optimization model for the shuttle bus charging schedule (1) postulates that all chargers are available anytime so that the shuttle bus can charge its battery whenever it wants. This assumption may be unrealistic, considering the availability of public charging stations. In Dastpak et al. (2024), the authors considered that charging stations are updated in real-time via binary Occupancy Indicator information signaling if a station is busy or not. Building on this, we adopt a scenario-based stochastic programming approach for the shuttle-bus charging schedule. Charger availability is represented by a finite set of scenarios sampled from this occupancy process. For each availability scenario, the optimization model determines $y_i, z_{ik\tau}, w_{ik}$, and p_{ik}, while x_{ij} is held fixed across all scenarios. In other words, the sequence of service trips is determined before the availability of the charger is known whereas the remaining decisions adapt to the realized availability of chargers. This is operationally reasonable, since a shuttle driver can decide when and where to charge based on the state of charge and the observed availability of chargers. The objective is to minimize the expected value of (1a) over the scenario distribution. A detailed formulation of the scenario-based model is provided in Appendix 3, and the computational time are reported in Table C.14.



(a) Availability 70% (b) Availability 80% (c) Availability 90%

Fig. 11. Total detour distance (kilometers).

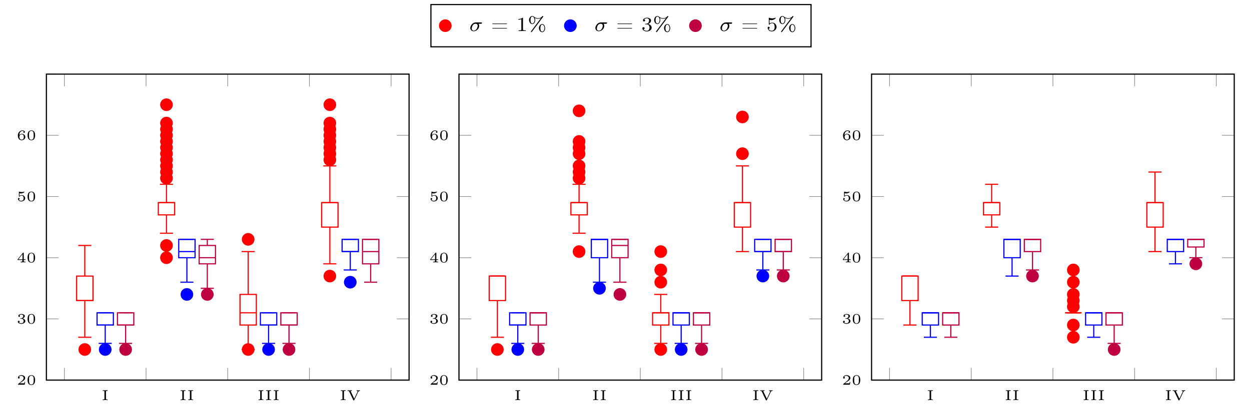


(a) Availability 70% (b) Availability 80% (c) Availability 90%

Fig. 12. Detour distance per charging station visit (kilometer/visit).

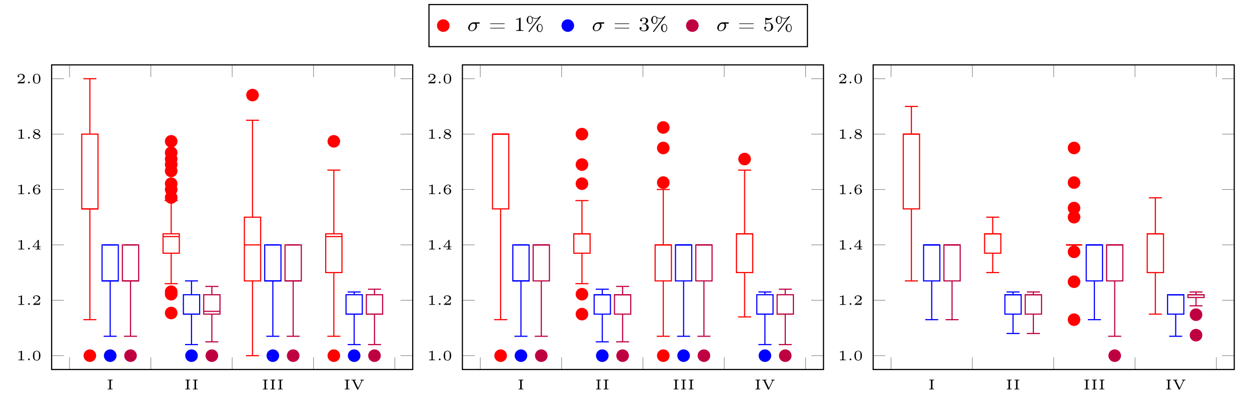
Table 8 reports the results of the shuttle bus charging schedules in terms of charging station visit, total charging amount in all service trips (kW), charging amount per charging station visit (kW), total detour traveling distance for visiting charging stations (km), detour traveling distance per charging station visit (km), total detour traveling time (minutes), and detour traveling time per charging station visit (minutes). The results of each item are reported in the following columns: "# visits", "charge amount", "charge per visit", "detour distance", "avg detour distance", "detour time", and "avg detour time", respectively. We consider five availabilities of the charger: 50%, 60%, 70%, 80%, and 90%. Also, three minimum charging ratio restrictions are considered: 1%, 3%, and 5%. We note that 16.667 kW is the maximum possible charging amount in a single service trip, which is 5.56% of the battery capacity B. Figs. 9–14 illustrate box plots of the results of each column in Table 8. Each figure consists of three subfigures, which consist of box plots for the different charging station availability of 70%, 80%, and 90%. Since there are no feasible charging schedules for regimes II and IV when the availability is 50% and 60%, box plots for the availability of 70%, 80%, and 90% are represented. In each subfigure, the x-axis represents a given regime. Each regime has three box plots, which represent the minimum charging amounts restricted to $0.01B, 0.03B$, and $0.05B$, respectively.

The results in Table 8 and Figs. 8–14 show that the number of charging station visits decreases when the availability of chargers increases. It implies that the shuttle bus must charge its battery whenever possible since the availability of chargers is low. Similarly, the number of charging station visits increases under regimes II and IV compared to regimes I and III. Since only 60% of the battery capacity is available under regimes II and IV, this results in more frequent charging compared to regimes I and III. For instance, the average charging station visit is around 27 under regime IV, which means that more than 75% of service trips are charging trips. A more frequent visit to charging stations results in a longer detour travel distance, leading to an increase in the total charging amount. Also, the shuttle bus tends to charge its battery more per visit when the availability of chargers is high. Therefore, higher public charging station availability can reduce inefficiency in shuttle bus operations.



(a) Availability 70% (b) Availability 80% (c) Availability 90%

Fig. 13. Total detour traveling time (minutes).



(a) Availability 70% (b) Availability 80% (c) Availability 90%

Fig. 14. Detour traveling time per charging station visit (minute/visit).

7. Conclusion

This paper investigates a pioneering framework considering electric shuttle vehicles with shared charging infrastructure and analyzes the impacts of the charging operations on passengers and the transit system. We specifically focus on commuting shuttle services with frequent headways and medium distance travels. We devise a charging scheduling optimization model and the corresponding passenger user equilibrium model to evaluate both the operational properties and behavioral impacts of electric shuttle services.

The scheduling optimization model aim to minimize the charging cost while satisfying the vehicle battery capacity and time feasibility. The model is tailored to multi-fleet operations with varying charging price. The model can also capture the different battery levels where battery degradation is considered. The results of the model yield the index of trips for charging activities, the shared charging station to be selected and the time of charging. In line with the charging scheduling output, a passenger onboarding user equilibrium model is further developed to analyze the effects of charging activities on number of passengers in vehicles and the transit system performance (e.g., queuing, crowding). A case study is conducted consider a commuting route connecting residential and central business area with references of Ontario region in Canada.

Results of the study show the charging activities can have important impacts on passengers' choices of departure times, and reshap the onboarding patterns where higher battery capacity with less charging trips can reduce queuing time and crowding during the peak period. This indicates the potential of integrating charging schedules with congestion management under the operations of electric shuttle with shared charging infrastructure. The developed study also assesses the impacts of time-variant electricity prices on charging schedules and passengers' onboarding patterns where peak-hour electricity pricing can result in more frequency charging trips for transit systems.

This study has made one of the first attempts to conceptualize electric shuttle with shared charging infrastructure and analyze the interactions of both operational conditions and passengers' behaviors. We focus on the daily operations of the electric transit and explore the optimal schedules of shuttle buses charging during the operations as well as the impacts on passengers' costs and on-boarding patterns. Under shared charging, buses can recharge during the day to maintain a sufficient state of charge, thereby reducing the need for overnight charging and the reliance on depot-based charging. The study opens new avenues to conduct cost-benefit analyses of utilizing and sharing charging infrastructure in public transit system. Future studies can be extended to explore economical analyses of long-term and capital planning of shared charging compared with depot charging. The study also has certain limitations. The analyses assumes homogeneous passengers with identical passenger preferences which may under- or over-estimate passengers' departure time choices and crowding. At the operational level, charging queue and joint optimizations are not considered (e.g., bus routes with charging schedules, charging costs with social welfare). Future research could examine different shuttle service types (e.g., school buses, campus bus, shuttles for elders), incorporate passenger heterogeneity in values of time, and investigate integrated optimization of bus operations and social welfare across diverse communities. Extensions to long-term planning can also be considered, including strategies that evaluate the investment and economic benefits of shared charging infrastructure for transit systems.

CRediT authorship contribution statement

Yili Tang: Writing – review & editing, Writing – original draft, Visualization, Project administration, Methodology, Funding acquisition, Formal analysis, Data curation, Conceptualization; **Bissan Ghaddar:** Writing – review & editing, Writing – original draft, Validation, Supervision, Project administration, Funding acquisition, Formal analysis, Conceptualization; **Jaehee Jeong:** Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Project administration, Methodology, Investigation, Formal analysis, Conceptualization; **Yi Wei:** Visualization.

Data availability

Data will be made available on request.

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Appendix A. Proof of the equilibrium equivalency

The feasible set $\mathcal{N} = \left\{\mathbf{n} \geq 0 : \sum_{i\in\Phi} n_i = N,\ n_i \leq S,\ \forall i \in \Phi\right\}$ is nonempty, closed, bounded, and convex, hence it is a compact set. The objective function Eq. (8a) is continuous on the compact set $\mathcal{N}$. Therefore, the optimization model has at least one minimum (Rudin, 1976; Sheffi, 1985), and hence a passenger user equilibrium exists. In addition, the objective function and the constraints of the optimization model Eqs. (8a)–(8d) are both strictly convex. The model thus has a unique solution.

The optmization model can be reformulated as a Lagrangian below

$$\min \quad L(\mathbf{n},\mathbf{u}) = \sum_{i\in\Phi} G(n_i)\cdot(t_i + \sum_{k\in K(i)} \eta_{ik} w_{ik} + \sum_{k\in K(i)} \delta_{ik} p_{ik}) + \sum_{i\in\Phi} n_i \cdot \zeta(a_i) \tag{A.1a}$$

$$+ \sum_{i\in\Phi} n_i \cdot \theta \cdot \lambda \cdot (\sum_{k\in K(i)} \eta_{ik} w_{ik} + \sum_{k\in K(i)} \delta_{ik} p_{ik}) + \sum_{i\in\Phi} \mu_i^2 \cdot (-S + n_i) + \mu^1 \cdot (N - \sum_{i\in\Phi} n_i) \tag{A.1b}$$

$$\text{s.t.} \quad n_i \geq 0, \qquad \forall i \in \Phi \tag{A.1c}$$

$$u^1 \geq 0, \tag{A.1d}$$

$$u_i^2 \geq 0, \qquad \forall i \in \Phi \tag{A.1e}$$

where u^1 and u_i^2 are the Lagrange multiplier. The solution of (A.1b)–(A.1e) can be obtained by solving the following first-order condition

$$n_i \cdot \frac{\partial L(n_i, u^1, u_i^2)}{\partial n_i} = 0 \text{ and } \frac{\partial L(n_i, u^1, u_i^2)}{\partial n_i} \geq 0, \qquad \forall i \in \Phi, \tag{A.2a}$$

$$\frac{\partial L(n_i, u^1, u_i^2)}{\partial u_1} \geq 0, \qquad \forall i \in \Phi, \tag{A.2b}$$

$$u_i^2 \cdot \frac{\partial L(n_i, u^1, u_i^2)}{\partial u_i^2} = 0 \text{ and } \frac{\partial L(n_i, u^1, u_i^2)}{\partial u_i^2} \leq 0 \qquad \forall i \in \Phi. \tag{A.2c}$$

Condition (A.2a) yield

$$\begin{cases} n_i\{c(a_i)+\zeta(a_i)+\theta\cdot\lambda\cdot(\sum_{k\in K(i)}\eta_{ik}w_{ik}+\sum_{k\in K(i)}\delta_{ik}p_{ik})+u_i^2-u^1\}=0, & \forall i\in\Phi \\ c(a_i)+\zeta(a_i)+\theta\cdot\lambda\cdot(\sum_{k\in K(i)}\eta_{ik}w_{ik}+\sum_{k\in K(i)}\delta_{ik}p_{ik})+u_i^2-u^1\geq 0, & \forall i\in\Phi \end{cases} \tag{A.3}$$

Condition (A.2b) yield

$$\begin{cases} N-\sum_{i\in\Phi}n_i=0 \\ u^1\geq 0 \end{cases} \tag{A.4}$$

Condition (A.2c) yield

$$\begin{cases} u_i^2\cdot(-S+n_i)=0, & \forall i\in\Phi \\ -S+n_i\leq 0, & \forall i\in\Phi \\ u_i^2\geq 0, & \forall i\in\Phi \end{cases} \tag{A.5}$$

Let u^1 be v and u_i^2 be $q(a_i)$, the above inequalities and equations can be expressed as

$$\begin{cases} n_i\{\zeta(a_i)+c(a_i)+q(a_i)+\theta\cdot\lambda\cdot(\sum_{k\in K(i)}\eta_{ik}w_{ik}+\sum_{k\in K(i)}\delta_{ik}p_{ik})-v\}=0, & \forall i\in\Phi \\ \zeta(a_i)+c(a_i)+q(a_i)+\theta\cdot\lambda\cdot(\sum_{k\in K(i)}\eta_{ik}w_{ik}+\sum_{k\in K(i)}\delta_{ik}p_{ik})\ -v\geq 0, \forall i\in\Phi \\ u_i^2\cdot(S-n_i)=0, & \forall i\in\Phi \\ S-n_i\geq 0, & \forall i\in\Phi \\ q(a_i)\geq 0, & \forall i\in\Phi \\ n_i\geq 0, & \forall i\in\Phi \\ N-\sum_{i\in\Phi}n_i=0 \\ v\geq 0 \end{cases} \tag{A.6}$$

This completes the proof.

Appendix B. Results of optimization model

Tables B.9–B.12 report the optimization model (1) results of shuttle bust #1 for regimes I–IV, respectively. For each service trip, we report departure time, detour time for visiting charger, charging time, amount and location, initial battery level when the shuttle starts the trip, and end time. The results regarding time are represented in hours, and energy-related results are reported in kWh. Detour and charging time are reported in minutes as mentioned in the tables.

Table B.9
Results - shuttle bus #1 of regime I.

σ = 1%	Trip index i	1	4	7	10	13	16	19	22	25	28	31	34
	Departure time	6:00 am	6:45 am	7:30 am	8:15 am	9:00 am	9:45 am	10:30 am	11:15 am	12:00 pm	12:45 pm	1:30 pm	2:15 pm
	Detour time (min.)	0	1	3	0	0	0	0	0	0	1	1	3
	Charging time (min.)	0	4	2	0	0	0	0	0	0	3.44	4	4.968
	Start battery level	270	244	233.967	215	189	163	137	111	85	59	46.633	36.6
	Charging amount		16.667	8.333							14.333	16.667	20.7
	Charging location		#2	#1							#2	#2	#1
	End time	6:40 am	7:30 am	8:15 am	8:55 am	9:40 am	10:25 am	11:10 am	11:55 am	12:40 pm	1:29 pm	2:15 pm	3:02 pm
σ = 3%	Trip index i	1	4	7	10	13	16	19	22	25	28	31	34
	Departure time	6:00 am	6:45 am	7:30 am	8:15 am	9:00 am	9:45 am	10:30 am	11:15 am	12:00 pm	12:45 pm	1:30 pm	2:15 pm
	Detour time (min.)	0	0	0	0	0	0	1	0	1	1	1	3
	Charging time (min.)	0	0	0	0	0	0	2.16	0	4	3.136	4	4.968
	Start battery level	270	244	218	192	166	140	114	96.3	70.3	60.267	46.633	36.6
	Charging amount							9		16.667	13.067	16.667	20.7
	Charging location							#2		#2	#2	#2	#1
	End time	6:40 am	7:25 am	8:10 am	8:55 am	9:40 am	10:25 am	11:13 am	11:55 am	12:45 pm	1:29 pm	2:15 pm	3:02 pm
σ = 5%	Trip index i	1	4	7	10	13	16	19	22	25	28	31	34
	Departure time	6:00 am	6:45 am	7:30 am	8:15 am	9:00 am	9:45 am	10:30 am	11:15 am	12:00 pm	12:45 pm	1:30 pm	2:15 pm
	Detour time (min.)	0	0	0	0	0	0	1	1	1	1	0	3
	Charging time (min.)	0	0	0	0	0	0	3.6	3.6	3.6	3.6	0	3.864
	Start battery level	270	244	218	192	166	140	114	102.3	90.6	78.9	67.2	41.2
	Charging amount							15	15	15	15		16.1
	Charging location							#2	#2	#2	#2		#1
	End time	6:40 am	7:25 am	8:10 am	8:55 am	9:40 am	10:25 am	11:14 am	11:59 am	12:44 pm	1:29 pm	2:10 pm	3:01 pm

Table B.10
Results - shuttle bus #1 of regime II.

σ = 1%	Trip index i	1	4	7	10	13	16	19	22	25	28	31	34
	Departure time	6:00 am	6:45 am	7:30 am	8:15 am	9:00 am	9:45 am	10:30 am	11:15 am	12:00 pm	12:45 pm	1:30 pm	2:15 pm
	Detour time (min.)	0	1	3	0	1	0	1	1	1	1	1	3
	Charging time (min.)	0	10.467	8.333	0	16.667	0	16.667	16.667	16.667	16.667	16.667	20.7
	Start battery level	240	214	197.767	178.8	152.8	142.767	116.767	106.733	96.7	86.667	76.633	66.6
	Charging amount		10.467	8.333		16.667		16.667	16.667	16.667	16.667	16.667	20.7
	Charging location		#2	#1		#2		#2	#2	#2	#2	#2	#1
	End time	6:40 am	7:28 am	8:15 am	8:55 am	9:45 am	10:25 am	11:15 am	12:00 pm	12:45 pm	1:30 pm	2:15 pm	3:02 pm
σ = 3%	Trip index i	1	4	7	10	13	16	19	22	25	28	31	34
	Departure time	6:00 am	6:45 am	7:30 am	8:15 am	9:00 am	9:45 am	10:30 am	11:15 am	12:00 pm	12:45 pm	1:30 pm	2:15 pm
	Detour time (min.)	0	1	1	0	1	1	1	1	0	1	1	3
	Charging time (min.)	0	4	4	0	4	4	4	2.16	0	2.208	4	4.968
	Start battery level	240	214	203.967	193.933	167.933	157.9	147.867	137.833	120.133	94.133	76.633	66.6
	Charging amount		16.667	16.667		16.667	16.667	16.667	9		9.2	16.667	20.7
	Charging location		#2	#2		#2	#2	#2	#2		#2	#2	#1
	End time	6:40 am	7:30 am	8:15 am	8:55 am	9:45 am	10:30 am	11:15 am	11:58 am	12:40 pm	1:28 pm	2:15 pm	3:02 pm
σ = 5%	Trip index i	1	4	7	10	13	16	19	22	25	28	31	34
	Departure time	6:00 am	6:45 am	7:30 am	8:15 am	9:00 am	9:45 am	10:30 am	11:15 am	12:00 pm	12:45 pm	1:30 pm	2:15 pm
	Detour time (min.)	0	0	0	1	1	1	1	1	1	1	1	3
	Charging time (min.)	0	0	0	3.6	3.6	3.6	3.6	3.6	3.6	3.6	3.6	4.536
	Start battery level	240	214	188	162	150.3	138.6	126.9	115.2	103.5	91.8	80.1	68.4
	Charging amount				15	15	15	15	15	15	15	15	18.9
	Charging location				#2	#2	#2	#2	#2	#2	#2	#2	#1
	End time	6:40 am	7:25 am	8:10 am	8:59 am	9:44 am	10:29 am	11:14 am	11:59 am	12:44 pm	1:29 pm	2:14 pm	3:02 pm

Table B.11
Results - shuttle bus #1 of regime III.

σ = 1%	Trip index i	1	4	7	10	13	16	19	22	25	28	31	34
	Departure time	6:00 am	6:45 am	7:30 am	8:15 am	9:00 am	9:45 am	10:30 am	11:15 am	12:00 pm	12:45 pm	1:30 pm	2:15 pm
	Detour time (min.)	1	1	1	0	0	0	0	0	1	0	0	3
	Charging time (min.)	2.784	4	4	0	0	0	0	0	2.512	0	0	4.968
	Start battery level	270	254.9	244.867	234.833	208.833	182.833	156.833	130.833	104.833	88.6	62.6	36.6
	Charging amount	11.6	16.667	16.667						10.467			20.7
	Charging location	#2	#2	#2						#2			#1
	End time	6:43 am	7:30 am	8:15 am	8:55 am	9:40 am	10:25 am	11:10 am	11:55 am	12:43 pm	1:25 pm	2:10 pm	3:02 pm
σ = 3%	Trip index i	1	4	7	10	13	16	19	22	25	28	31	34
	Departure time	6:00 am	6:45 am	7:30 am	8:15 am	9:00 am	9:45 am	10:30 am	11:15 am	12:00 pm	12:45 pm	1:30 pm	2:15 pm
	Detour time (min.)	1	1	1	0	1	0	0	0	0	0	0	3
	Charging time (min.)	2.784	4	4	0	2.512	0	0	0	0	0	0	4.968
	Start battery level	270	254.9	244.867	234.833	208.833	192.6	166.6	140.6	114.6	88.6	62.6	36.6
	Charging amount	11.6	16.667	16.667		10.467							20.7
	Charging location	#2	#2	#2		#2							#1
	End time	6:43 am	7:30 am	8:15 am	8:55 am	9:43 am	10:25 am	11:10 am	11:55 am	12:40 pm	1:25 pm	2:10 pm	3:02 pm
σ = 5%	Trip index i	1	4	7	10	13	16	19	22	25	28	31	34
	Departure time	6:00 am	6:45 am	7:30 am	8:15 am	9:00 am	9:45 am	10:30 am	11:15 am	12:00 pm	12:45 pm	1:30 pm	2:15 pm
	Detour time (min.)	1	1	1	0	0	1	0	0	0	0	0	3
	Charging time (min.)	3.6	3.92	4	0	0	3.6	0	0	0	0	0	3.6
	Start battery level	268.4	256.4	246.033	236	210	184	172.3	146.3	120.3	94.3	68.3	42.3
	Charging amount	15	16.333	16.667			15						15
	Charging location	#3	#2	#2			#2						#1
	End time	6:44 am	7:29 am	8:15 am	8:55 am	9:40 am	10:29 am	11:10 am	11:55 am	12:40 pm	1:25 pm	2:10 pm	3:01 pm

Table B.12
Results - shuttle bus #1 of regime IV.

σ = 1%	Trip index i	1	4	7	10	13	16	19	22	25	28	31	34
	Departure time	6:00 am	6:45 am	7:30 am	8:15 am	9:00 am	9:45 am	10:30 am	11:15 am	12:00 pm	12:45 pm	1:30 pm	2:15 pm
	Detour time (min.)	1	1	1	1	1	1	1	3	0	0	0	3
	Charging time (min.)	2.784	4	4	3.728	4	4	4	2	0	0	0	4.968
	Start battery level	240	224.9	214.867	204.833	193.667	183.633	173.6	163.567	144.6	118.6	92.6	66.6
	Charging amount	11.6	16.667	16.667	15.533	16.667	16.667	16.667	8.333				20.7
	Charging location	#2	#2	#2	#2	#2	#2	#2	#1				#1
	End time	6:43 am	7:30 am	8:15 am	8:59 am	9:45 am	10:30 am	11:15 am	12:00 pm	12:40 pm	1:25 pm	2:10 pm	3:02 pm
σ = 3%	Trip index i	1	4	7	10	13	16	19	22	25	28	31	34
	Departure time	6:00 am	6:45 am	7:30 am	8:15 am	9:00 am	9:45 am	10:30 am	11:15 am	12:00 pm	12:45 pm	1:30 pm	2:15 pm
	Detour time (min.)	1	1	1	0	1	1	0	1	1	0	1	3
	Charging time (min.)	2.784	4	4	0	2.16	4	0	4	4	0	3.424	4.968
	Start battery level	240	224.9	214.867	204.833	178.833	161.133	151.1	125.1	115.067	105.033	79.033	66.6
	Charging amount	11.6	16.667	16.667		9	16.667		16.667	16.667		14.267	20.7
	Charging location	#2	#2	#2		#2	#2		#2	#2		#2	#1
	End time	6:43 am	7:30 am	8:15 am	8:55 am	9:43 am	10:30 am	11:10 am	12:00 pm	12:45 pm	1:25 pm	2:14 pm	3:02 pm
σ = 5%	Trip index i	1	4	7	10	13	16	19	22	25	28	31	34
	Departure time	6:00 am	6:45 am	7:30 am	8:15 am	9:00 am	9:45 am	10:30 am	11:15 am	12:00 pm	12:45 pm	1:30 pm	2:15 pm
	Detour time (min.)	1	1	1	0	1	0	0	1	1	1	1	3
	Charging time (min.)	3.6	4	4	0	3.6	0	0	3.6	3.6	3.6	3.6	4.192
	Start battery level	238.4	226.4	216.367	206.333	180.333	168.633	142.633	116.633	104.933	93.233	81.533	69.833
	Charging amount	15	16.667	16.667		15			15	15	15	15	17.467
	Charging location	#3	#2	#2		#2			#2	#2	#2	#2	#1
	End time	6:44 am	7:30 am	8:15 am	8:55 am	9:44 am	10:25 am	11:10 am	11:59 am	12:44 pm	1:29 pm	2:14 pm	3:02 pm

Appendix C. Scenario-based optimization model

Model (C.1) is a mathematical formulation of a scenario-based optimization model that determines the charging schedule given the availability of chargers. Table C.13 presents additional notation for scenario-based optimization model (C.1). The computational time is given in Table C.14.

Table C.13
Additional notation for the scenario-based optimization model.

Sets	
$\mathcal{N}_s$	Set of scenarios of the availability of chargers
Variables	
y_i^{sc}	Nonnegative continuous decision variable for the remaining battery energy level at the beginning of the service trip i (kWh) in scenario sc
$z_{ik\tau}^{sc}$	Binary decision variable for k-th recharging position for the service trip i at time bucket τ in scenario sc. If a bus of trip i arrives to k-th recharging position at time bucket τ in scenario sc, then $z_{ikt}^{sc} = 1$, and 0 otherwise
w_{ik}^{sc}	Binary decision variable for k-th recharging position for the service trip i in scenario sc. If a bus of trip i visits k-th recharging position in scenario sc, $w_{ik}^{sc} = 1$, and 0 otherwise
p_{ik}^{sc}	Nonnegative continuous decision variable for the recharging amount for the k-th recharging position during the service trip i (kWh) in scenario sc
Parameters	
ξ_{ik}^{sc}	If the charger located at the k-th recharging position during the service trip i is available in scenario sc, $\xi_{ik}^{sc} = 1$, and 0 otherwise.

$$\min \sum_{i\in\Phi} \sum_{k\in K(i)} \sum_{t\in T} \sum_{sc\in\mathcal{N}_s} c_{ikt} z_{ikt}^{sc} p_{ik}^{sc} \tag{C.1a}$$

$$\text{s.t.} \sum_{i\in\Phi} x_{0i} \le v \tag{C.1b}$$

$$\sum_{j\in\Phi_{Q+1}, i\neq j} x_{ij} = 1 \qquad \forall i \in \Phi, \tag{C.1c}$$

$$\sum_{i\in\Phi_0, i\neq j} x_{ij} - \sum_{i\in\Phi_{Q+1}, i\neq j} x_{ji} = 0 \qquad \forall j \in \Phi, \tag{C.1d}$$

$$\sum_{t\in T} z_{ikt}^{sc} = w_{ik}^{sc} \qquad \forall k \in K(i), i \in \Phi, sc \in \mathcal{N}_s \tag{C.1e}$$

$$\sigma B w_{ik}^{sc} \xi_{ik}^{sc} \leq p_{ik}^{s} c \leq B w_{ik}^{sc} \xi_{ik}^{sc} \quad \forall k \in K(i), i \in \Phi, sc \in \mathcal{N}_s \tag{C.1f}$$

$$a_i + t_i x_{ij} + \sum_{k \in K(i)} \left(\eta_{ik} w_{ik}^{sc} + \delta_{ik} p_{ik}^{sc} \right) - \bar{M}(1 - x_{ij}) \leq a_j \quad \forall i \in \Phi_0, j \in \Phi_{Q+1}, i \neq j, sc \in \mathcal{N}_s \tag{C.1g}$$

$$a_i + t_i x_{i,Q+1} + \sum_{k \in K(i)} \left(\eta_{ik} w_{ik}^{sc} + \delta_{ik} p_{ik}^{sc} \right) - \bar{M}(1 - x_{i,Q+1}) \leq \hat{T} \quad \forall i \in \Phi, sc \in \mathcal{N}_s \tag{C.1h}$$

$$y_j^{sc} \leq y_i^{sc} - h_{ij} x_{ij} + \sum_{k \in K(i)} \left(p_{ik}^{sc} - \rho_{ik} w_{ik}^{sc} \right) + B(1 - x_{ij}) \quad \forall i \in \Phi, j \in \Phi_{Q+1}, i \neq j, sc \in \mathcal{N}_s \tag{C.1i}$$

$$y_i^{sc} + \sum_{\kappa=0}^{k-1} \left(p_{i\kappa}^{sc} - \beta_{i\kappa} - \rho_{i\kappa} w_{i\kappa}^{sc} \right) - \beta_{ik} - \hat{\rho}_{ik} w_{ik}^{sc} \geq \hat{B}_1 \quad \forall k \in K(i), i \in \Phi, sc \in \mathcal{N}_s \tag{C.1j}$$

$$y_i^{sc} + \sum_{\kappa=0}^{k-1} \left(p_{i\kappa}^{sc} - \beta_{i\kappa} - \rho_{i\kappa} w_{i\kappa}^{sc} \right) + p_{ik}^{sc} - \beta_{ik} - \hat{\rho}_{ik} w_{ik}^{sc} \leq \hat{B}_2 \quad \forall k \in K(i), i \in \Phi, sc \in \mathcal{N}_s \tag{C.1k}$$

$$y_i^{sc} + \sum_{k \in K(i)} \left(p_{ik}^{sc} - \beta_{ik} - \rho_{ik} w_{ik}^{sc} \right) \leq \hat{B}_2 \quad \forall i \in \Phi, sc \in \mathcal{N}_s \tag{C.1l}$$

$$(\tau - 1)\Delta - \bar{M}(1 - z_{ik\tau}^{sc}) \leq a_i + \sum_{\kappa=0}^{k-1} \left(\gamma_{i\kappa} + \delta_{i\kappa} p_{i\kappa}^{sc} + \eta_{i\kappa} w_{i\kappa}^{sc} \right) + \gamma_{ik} + \hat{\eta}_{ik} w_{ik}^{sc} \quad \forall k \in K(i), i \in \Phi, \tau \in T, sc \in \mathcal{N}_s \tag{C.1m}$$

$$a_i + \sum_{\kappa=0}^{k-1} \left(\gamma_{i\kappa} + \delta_{i\kappa} p_{i\kappa}^{sc} + \eta_{i\kappa} w_{\kappa}^{sc} \right) + \gamma_{ik} + \hat{\eta}_{ik} w_{ik}^{sc} \leq \tau\Delta + \bar{M}(1 - z_{ik\tau}^{sc}) \quad \forall k \in K(i), i \in \Phi, \tau \in T, sc \in \mathcal{N}_s \tag{C.1n}$$

$$x_{ij} \in \{0, 1\} \quad \forall i \in \Phi_0, j \in \Phi_{Q+1}, i \neq j, \tag{C.1o}$$

$$z_{ikt}^{sc} \in \{0, 1\} \quad \forall k \in K(i), i \in \Phi, \tau \in T, sc \in \mathcal{N}_s \tag{C.1p}$$

$$w_{ik}^{sc} \in \{0, 1\} \quad \forall k \in K(i), i \in \Phi, sc \in \mathcal{N}_s \tag{C.1q}$$

$$p_{ik}^{sc} \geq 0 \quad \forall k \in K(i), i \in \Phi, sc \in \mathcal{N}_s \tag{C.1r}$$

$$y_i^{sc} \geq 0 \quad \forall i \in \Phi, sc \in \mathcal{N}_s. \tag{C.1s}$$

Table C.14
CPU times (s) of scenario-based optimization model.

Availability	σ	Regime I	Regime II	Regime III	Regime IV
50%	1%	2841	–	2334	–
	3%	2578	–	2810	–
	5%	2198	–	2587	–
60%	1%	6995	564	5815	5967
	3%	155	–	1,136	–
	5%	1717	–	96	–
70%	1%	11,531	580	2206	559
	3%	398	511	114	3765
	5%	292	4082	321	4287
80%	1%	638	904	369	658
	3%	336	768	151	264
	5%	394	418	257	266
90%	1%	681	1119	208	342
	3%	328	2592	112	161
	5%	411	568	145	198

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