Accepted Manuscript

DOI: https://doi.org/10.1080/00207543.2021.1931725

Citation: Jinpeng Xu & Yufei Huang & Emmanouil Avgerinos & Gengzhong Feng & Feng Chu, 2022. "Dual-channel competition: the role of quality improvement and price-matching," International Journal of Production Research, Taylor & Francis Journals, vol. 60(12), pages 3705-3727, June.

Dual-channel Competition: The Role of Quality Improvement and Price-Matching

Quality improvement and price-matching are two commonly used competing strategies by the retailers. However, it is still unclear how the retailers should deliberate over the two strategies when selling in both online and offline markets. In this paper, we consider two dual-channel retailers selling a substitutable product to consumers in both online and offline markets. Especially, the retailers compete in the online market, and their offline markets are exclusive to themselves. We establish a game-theoretical model to investigate the trade-off between quality improvement and price-matching in competition, and the impact on retailers' profits and consumer surplus in the dual-channel market structure. The analysis shows that, first, a retailer should choose to improve its quality to avoid price competition when the online market is small; second, when retailers engage in price competition, the retailer with larger offline market is more willing to adopt price-matching, while the retailer with a small share of offline market can be hurt; third, quality improvement can always increase the consumer surplus, while price-matching always hurts consumer surplus due to price collusion.

Key words: price-matching; quality improvement; dual-channel retailing; market structure; game theory

1. Introduction

The surge of the Internet economy has encouraged many retailers to open online shops in addition to their physical stores, and sell the products via both online and offline channels. While dual-channel retailing can serve different consumer segments and make shopping more convenient for consumers, it also engenders new challenges to retailers in competition. With more purchase channel options, consumers become more savvy in their purchasing behavior. They can compare prices between different channels and across different retailers (Neslin et al. 2006; Kireyev et al. 2017), research products in physical stores and then buy online (Gao & Su 2016, 2017). Considering this, retailers need to carefully revise and devise their strategies in dual-channel competition.

Pricing is a common and important tool that retailers use in the competition, and at the same time, price-matching policy is reported to be an effective marketing strategy to increase sales, especially in online retailing (Lazar 2020). The price-matching policy, also called lowest price guarantee policy, is provided by many retailers, such as USA's Target, France's Carrefour, UK's Tesco, and China's Jing Dong mall, among others. When one retailer offers price-matching policy, consumers who buy a product from this retailer are promised to be paid the difference if they find a lower price in the market. Such a price-matching strategy can be applied to self-matching between a retailer's different selling channels (Kireyev et al. 2017) or different selling periods (Lai et al. 2010). A retailer can also apply price-matching by referring to its competitor's price. In this case, such a strategy is also called competitive price-matching. In a dual-channel scenario, a retailer should consider its competitor's pricing strategy to decide the prices for its own channels, as well as whether to match the competitor's price.

Although price-matching policy may help enlarge market share and is adopted widely, it is observed that some dual-channel retailers only offer the policy during sale events, and some others even ended the policy in practice. For example, in China, Gome and Suning, two large retailers operating both online and offline retail channels, only provide price-matching policy during some online sale events, such as the mid-year sale in June and the Double-Eleven sale in November. In the US, Walmart ended its price-matching policy in 2019, and some other retailers such as Target, still offer this policy. In the UK, a large dual-channel store, Sainsbury's, introduced price-matching guarantees in 2011 but stopped offering them in 2016 (Butler 2016). Similarly, a high-end groceries store, Waitrose, began matching the prices of UK's largest groceries store Tesco in 2010. However, in November 2018, it drastically scaled down the price-matching scheme and started to differentiate the upmarket grocery products with higher quality in an attempt to rebuild profits (Chambers 2018). According to Information Resources Inc. (2018), there exists a trend in recent years that retailers have invested heavily to provide differentiated products with higher quality, rather than selling products identical to competitors.

From these examples, we find two observations. First, a retailer may choose different strategies in competition. It can either choose price-matching to compete directly via pricing to gain more market share, or choose to improve its quality instead to avoid direct price competition. By offering high-quality products, a retailer can distinguish itself from the competitors, avoid direct competition and gain more profit from the market. Second, the retailer's deliberation on the strategies, such as price-matching or quality improvement, may be impacted by the online and offline market share. The expand of online market may drive the retailer to change its strategy in competition. While both price-matching and quality improvement are used in practice, few studies have explained how and why retailers choose between these two alternative strategies in a dual-channel setting, and it is unclear how the size of a retailer's online and offline market can impact its decision. Our paper is motivated by these observations and aims to examine how and why retailers choose price-matching or quality improvement strategy in a dual-channel setting.

To address these questions, we develop a game-theoretical model to capture the competitive dual-channel setting. Consider that there are two retailers and the market is segmented into three parts, an exclusive offline market for each retailer, and an online market in which the two retailers compete with each other. In the market, rational consumers make purchase decisions to maximize their individual utility. Different size of the offline and online market may change a retailer's focus between the offline and online market, and consequently, impact the retailer's optimal strategy in competition. Therefore, we study when and why dual-channel retailers choose quality improvement or price-matching in competition, and how the dual-channel market structure impacts retailers' strategies, and the consequent profits and consumer surplus. Based on the ex-ante market structure, we first study the retailers' equilibrium decisions, and then examine how the dual-channel market structure influences their decisions and how such decisions affects their profits and consumer surplus.

The analysis shows, first, a retailer chooses to improve its quality to avoid price competition when the online market is small. Quality improvement can always increase the retailer's profit and consumer surplus, while decrease the competitor's profit. Second, if the retailers engage in price competition, offering price-matching is not always optimal for retailers in equilibrium. In this case, the retailer with a large exclusive offline market is more willing to adopt price-matching, while the other retailer with a small exclusive offline market is more inclined to not offer price-matching, especially when the size of the competitive online market is small. Finally, price-matching can lead to price collusion and can thus hurt the consumer surplus. Overall, the main contribution of our paper is that, we find the dual-channel market structure impacts how the retailers choose price-matching or quality improvement strategy. The findings can explain when and why retailers should choose quality improvement or price-matching, and help retailers devise their optimal strategies in competition.

The rest of the paper is organized as follows. In §2, we provide a review of the related literature. In §3, the problem is described and retailer price and quality decisions in context of dual-channel competition are summarized. In §4, a game-theoretical model is established to analyze each retailer decision and provide the optimal strategy for each decision, and management insights are discussed. We conclude and discuss future research directions in §5.

2. Literature Review

This paper is mainly related to three streams of research, dual-channel retailing, price-matching guarantees, and quality competition. We next discuss these three streams, respectively.

With the surge of e-commerce and the Internet economy, many retailers have introduced an online selling channel in addition to their traditional physical stores. This trend has significantly changed both consumer's purchasing behavior and retailer's selling behavior (Lal & Sarvary 1999). For consumers, the existence of multiple channels enables them to compare prices (Verhoef et al. 2007) and to gather product information from one channel and buy in another channel (Gao & Su 2017). Consumers can also consider the product reviews generated by buyers online, when making purchasing decisions (Chevalier & Mayzlin 2006). In such a multichannel retailing setting, the existing literature has mainly focused on pricing (Chiang et al. 2003), inventory management (Gallino & Moreno 2014), information strategy (Zettelmeyer 2000), consumer returns (Ofek et al. 2011), and channel synergy strategy (Zhang et al. 2019). Furthermore, Neslin et al. (2006) and Neslin and Shankar (2009) discuss how to conduct multichannel customer management. Balasubramanian (1998) studies how online retailers compete with offline retailers, and Biyalogorsky and Naik (2003) study how a retailer can manage its own multiple channels. Ofek et al. (2011) investigate the timing of setting up an online channel. Konuş et al. (2008) empirically study how multiple channels influence market segmentation and show that retailers not only need to carefully manage their own retail channels but also need to consider their competitor's channel strategies. In addition, there are a few papers examine channel and pricing strategy (Xu et al. 2020; He et al. 2019), inventory and quality decision (He et al. 2020; Zhang et al. 2020), and service strategy (Lin et al. 2020; He et al. 2014; Li et al. 2019; Dan et al. 2018) in a dual-channel supply chain. The existing literature usually focuses on the decision of introducing the online channel, analyzes the impact of dual-channel retailing on price, inventory and consumer behavior, and investigates channel and service strategy in multi-channel supply chains. However, in a competitive environment, how dual-channel retailers choose between different strategies, such as price-matching and quality improvement, to maximize their profit has not been well examined.

There is also rich literature on various price-matching guarantees. When price-matching guarantees match the lower price from a retailer itself, they are noncompetitive price-matching or internal price-matching. In addition, when price-matching guarantees match the lower price from a retailer's competitor, they are usually referred to as competitive price-matching. The literature focusing on noncompetitive price-matching mainly examines how retailers can design and use these guarantees to influence consumer behaviors and make more profits (Butz 1990; Png 1991; Levin et al. 2007; Lai et al. 2010; Xu 2011). For competitive pricing-matching, the existing literature mainly focuses on whether competing retailers should adopt price-matching guarantees and how to set optimal prices in equilibrium (Corts 1997; Dana Jr 2001; Deneckere & Peck 1995). An interesting finding is that competitive price-matching guarantees can lead to price collusion, and retailers will rise prices in equilibrium (Hay 1982; Salop 1986). In addition, price collusion exists no matter whether retailers determine prices simultaneously or sequentially (Belton 1987; Sargent 1992) or whether the market is duopolistic or oligopolic (Corts 1995; Doyle 1988). Competitive price-matching guarantees can also be used to differentiate consumers (Baye & Kovenock 1994; Edlin 1997; Png & Hirshleifer 1987) or to signal the cost and quality of the retailer in competition (Moorthy & Winter 2006; Moorthy & Zhang 2006). Nalca et al. (2010, 2013) introduce inventory availability verification clause and study the way in which such a clause affects competing retailers' decisions regarding price-matching guarantees. There is limited literature that studies price-matching in the dual-channel setting, which is more common in practice. Recently, Nalca (2017) considers a dual-channel setting in which the supplier can sell via a retailer or directly to the consumers, and further studies the supplier's and the retailer's pricing and price-matching decisions. However, when two dual-channel retailers compete, how different market structures impact the retailers' decisions regarding the adoption of price-matching policy has not been adequately investigated.

Our research is also related to the literature on quality decision and competition. Quality decision is studied in the context of outsourcing (Bae et al. 2010), dynamic learning and selling (Keskin & Birge 2019), marketing strategy "pay what you want" (Schmidt et al. 2015), supply chain coordination (Gao et al. 2016) and facility location (Saidani et al. 2012; Wu et al. 2020). A few researches investigate the equilibrium on quality and price of competing firms with different costs (Chambers et al. 2006; Jing 2006) and under yield uncertainty (Federgruen & Yang 2009). There are also some researches address the price and quality competition in product entry and new product introduction, often assuming that the later entrant or the new product comes with higher quality due to technology vintage effects. Among them, Bohlmann et al. (2002) propose a product entry model that incorporates consumer valuations of product variety and quality to study the advantages and disadvantages of the pioneer and the later entrant, and demonstrate how vintage effects can help the later entrant dominate the market in the competition. Klastorin and Tsai (2004) consider two firms begin to develop a new product simultaneously and study the equilibrium on product design level and pricing. Wang and Hui (2017) investigate the product introduction and pricing problem in the life span of information technology products between an increment and an entrant. Huang et al. (2018) study how an entrepreneurial firm with a high-quality product should compete with the incumbent firm by considering consumer peer learning and incumbent reaction. Rao and Turut (2019) examine the new product preannouncement strategy under competition. In general, the existing literature focuses on the quality and pricing decision in various competition contexts. However, few researches compare the role of the two commonly used strategies, quality improvement and price-matching, in the context of dual-channel competition.

Different from existing literature, to the best of our knowledge, this paper is one of the first studies examining the retailer's deliberation over price-matching and quality improvement in dual-channel competition. We also consider the two retailers' heterogeneous market share in both the offline and online market. We therefore shed light on the impact of the market structure on retailers' equilibrium decision regarding price-matching and quality improvement, and analyze how different strategies impact the retailers' profits and consumer surplus.

3. Problem Description

Consider two retailers, indexed by $i \in \{1, 2\}$, selling substitutable products to rational consumers in dual channels, offline market and online market. Each retailer has a unit product selling cost c_i. The retailers have their own exclusive offline markets, in which consumers are loyal and buy products only from the respective retailers, and the retailers compete in the online market. Similar to Narasimhan (1988), the market size is normalized to 1, and the ratio of the online market to the total market is β, where $\beta \in [0,1]$. For the offline market, the ratio of retailer 1's offline market to the total offline market is α, where $\alpha \in [0,1]$. Therefore, the size of the exclusive offline market is $\alpha(1-\beta)$ for retailer 1 and $(1-\alpha)(1-\beta)$ for retailer 2, and the size of the competitive online market is β.

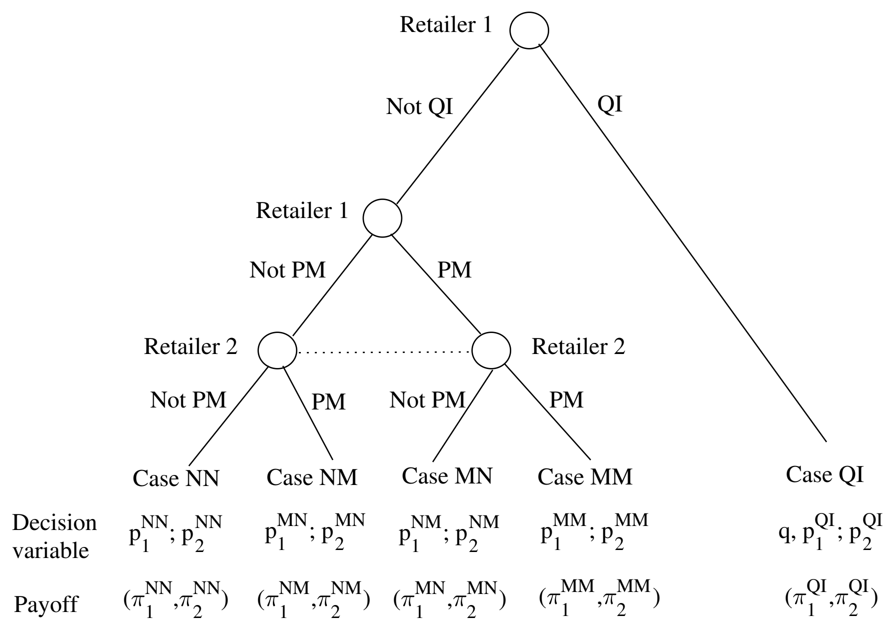
In this paper, we use the Hotelling model to capture the competition of the two retailers in the market, which is widely used in the literature (e.g., Ofek et al. 2011; Kireyev et al. 2017). We consider the two retailers are located at the two ends of a unit Hotelling line and consumers are uniformly distributed on the line. The consumers have heterogeneous preferences over the two retailers, and such heterogeneity in preference is represented by the disutility that consumers incur when they buy from one of the retailers. The disutility of online consumers is t per unit distance, so that an online consumer, located at $x \in [0,1]$ on the unit line, incurs the disutility of xt or $(1-x)t$ when buying from retailer 1 or retailer 2, respectively. Since the offline market is exclusive to the corresponding retailer, the disutility for offline consumers is assumed to be zero when these offline consumers visit their local retailer and is infinite when they visit the other retailer. As online consumers are usually better informed than offline consumers and make purchasing decisions by comparing the two retailers and maximizing their utilities, resulting in higher valuation for the same product (Hsiao & Chen 2014), we assume the online consumers' valuation to 1, and offline consumers' valuation on the product is $d < 1$. [1]

In the competition, if both retailers sell the same product, they will engage in direct price competition in which they make decisions on whether to offer price-matching or not in a Nash game. A retailer can also choose quality improvement as its strategy to avoid direct price competition with the competitor. For example, a retailer can add additional features, change package designs and provide value-added services to improve the product quality (Jerath et al. 2017), which results in a differentiation from its competitor's product and

[1] For ease of exposition, we assume that the valuation of the consumers in offline market d is neither too large nor too small, i.e., $t+\max(c_1,c_2) \leq d \leq \min(1-\frac{t}{2}, \frac{3t}{2}+\frac{c_1+c_2}{2}) < 1$, which helps to rule out the trivial cases, ensuring that the whole market can be fully covered and the two retailers always compete in the online market. More specifically, $d \geq t+\max(c_1,c_2)$ ensures that the offline market will not be abandoned, and $d \leq \min(1-\frac{t}{2}, \frac{3t}{2}+\frac{c_1+c_2}{2})$ ensures that the online market can always be covered and both retailers have a positive market share in equilibrium. Similar assumption can be found in the literature (e.g., Ofek et al. 2011; Kireyev et al. 2017).

an increase in consumers' willingness-to-pay for its product (Rao & Turut 2019). However, quality improvement decision for the retailer is usually not instant and cannot be easily matched by the competitor (Huang et al. 2018). To acquire the ability to improve product quality, a retailer may need to carry out a long-term investment. For model simplicity and traceability, we assume only retailer 1 can improve the product quality, and choose it as the focal retailer to investigate its optimal strategy choosing between quality improvement and price-matching. This allows us to analytically solve for the retailers' optimal strategy in closed-from.[2]

Figure 1 Sequence of Events.



The sequence of events is described in Fig. 1. First, the focal retailer 1 decides whether to provide the same product as retailer 2 or adopt quality improvement (QI) to offer a product with higher quality, which results in an extra cost but can avoid direct price competition. Second, if choosing quality improvement, retailer 1 announces its strategy and decides on the quality improvement level and the retail price, while retailer 2 only decides on its retail price; otherwise, if retailer 1 decides to offer the same product as retailer 2, the two retailers further decide whether to offer price-matching (PM) to consumers, and then set their retail

[2] The approach of focusing on a focal retailer's behavior is commonly used to analyze complicated behaviors in retail competition, see (Ishida & Taylor 2012) for an example. We will discuss the case that retailer 2 can also improve the product quality using numerical analysis. These results are summarized in Fig.6.

prices. Finally, the consumers observe the two retailers' strategies, product quality and retailing prices, and make purchase decisions. In our paper, when price-matching is offered by a retailer, if the other retailer's price is lower, the price gap will be refunded to the consumers. Also, the retailers apply the same price to both its online and offline channels, and the price-matching policy is available for all online and offline consumers.

We use the superscript QI to denote the case of quality improvement and use NN to denote the case in which the price-matching policy is not offered by the two retailers. Similarly, we use MN, NM, and MM to denote the cases in which price-matching is offered by retailer 1, retailer 2, or both retailers, respectively. A summary of all notations is provided in Table 1.

Table 1 Summary of Notation

Notation	Explanation
i	Index for retailers, $i \in \{1,2\}$
k	Index for different cases, $k \in \{NN, MN, MN, MM, E, QI\}$
NN	Superscript for the base case without price-matching or quality improvement
MN	Superscript for the case when price-matching is only offered by retailer 1
NM	Superscript for the case when price-matching is only offered by retailer 2
MM	Superscript for the case when price-matching is offered by the both retailers
E	Superscript for the equilibrium outcome of price-matching
QI	Superscript for the case when retailer 1 adopts quality improvement strategy
c_i	Retailer i's selling cost
p_i^k	Retailer i's retail price in case k
π_i^k	Retailer i's profit in case k
α	Retailer 1's ratio of offline market to the total offline market
β	Ratio of online market to the total market
d	Offline consumers' valuation of the product
t	Measure for online consumers' preference of the retailers
x	Consumers' preference location, $x \in [0,1]$
u_i	Consumers' utility when purchasing from retailer i
q	Retailer 1's quality improvement level under QI strategy
$c(q)$	Retailer 1's cost of quality improvement under QI strategy
$u(q)$	Consumers' additional utility of retailer 1's products under QI strategy

4. Analysis of Optimal Strategy

In this section, we start with the base case NN without price-matching and quality improvement, and analyze the retailers' equilibrium retail prices and the resulting profits. We then proceed to solve for the cases when one or both retailers adopt price-matching, i.e., case MN, NM and MM, and the case when the focal retailer 1 implements quality improvement, i.e., case QI, respectively. We finally compare the results from these cases with the base case to investigate the impact of price-matching or quality improvement on the retailers' prices and profits, as well as consumer surplus. In each case, we first give the profit functions of the retailers based on the Hotelling model and then derive the optimal price or quality decisions. All proofs are provided in the Appendix.

4.1. Base Case (Case NN)

In this case, no retailer adopts quality improvement or price matching strategy. For example, some dual-channel retailers, such as Walmart and Safeway, sell many identical products and do not offer price-matching policy. The retailers simultaneously decide on their retail prices p_1^{NN} and p_2^{NN} in the competition. Given the retailers' prices, while consumers in the offline market will buy from the corresponding retailer i if her valuation is larger than the retail price, $d > p_i^{NN}$, consumers in the online market make purchase decisions by comparing the two retailers and maximizing their utility. Specifically, if an online consumer located at x buys from retailer 1, her utility is $u_1 = 1 - p_1^{NN} - tx$, and if she buys from retailer 2, her utility is $u_2 = 1 - p_2^{NN} - t(1-x)$. To find the equilibrium, by letting $u_1 = u_2$ and solving for x, a consumer located at $\hat{x} = \frac{p_2^{NN} - p_1^{NN} + t}{2t}$ is indifferent between buying from retailer 1 and retailer 2. Therefore, online consumers with $x \leq \hat{x}$ will buy from retailer 1, and those with $x > \hat{x}$ will buy from retailer 2. By considering the size of the offline and online market, the total market share for retailer 1 and retailer 2 can be written as $\alpha(1-\beta) + \beta\frac{p_2^{NN} - p_1^{NN} + t}{2t}$ and $(1-\alpha)(1-\beta) + \beta\frac{p_1^{NN} - p_2^{NN} + t}{2t}$, respectively. Consequently, the two retailers' total profits can be written as,

$$\max_{p_1^{NN}} \pi_1^{NN} = \big(\alpha(1-\beta) + \beta\frac{p_2^{NN} - p_1^{NN} + t}{2t}\big)(p_1^{NN} - c_1), \tag{1}$$

$$\max_{p_2^{NN}} \pi_2^{NN} = \big((1-\alpha)(1-\beta) + \beta\frac{p_1^{NN} - p_2^{NN} + t}{2t}\big)(p_2^{NN} - c_2). \tag{2}$$

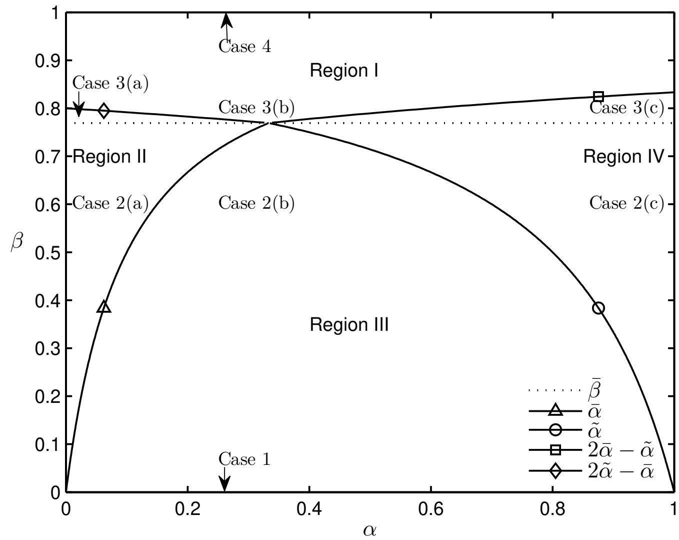
To obtain the two retailers' optimal prices and profits in equilibrium, we need to solve the retailers' profit maximization problems, (1) and (2). Due to the complexity of the model, different values of α and β can lead to different interior or corner solutions of the equilibrium prices. It is straightforward to solve the cases when the whole market is offline (i.e., $\beta = 0$) or online (i.e., $\beta = 1$). However, for $\beta \in (0,1)$, the equilibrium outcome will differ for small or large online market (i.e., the value of β is low or high) and small, medium or large offline market (i.e., the value of α is low, medium or high). As a result, we solve in total eight cases, and for each case, we study how the online and offline market size impact a retailer's optimal price and profit. These results are summarized in Proposition 1.

PROPOSITION 1. *When the retailers do not offer price-matching, the optimal retail prices and the profits are as follows:*

1. *if* $\beta = 0$*, then*
$$p_1^{NN} = p_2^{NN} = d,$$
$$\pi_1^{NN} = \alpha(d - c_1), \quad \pi_2^{NN} = (1-\alpha)(d - c_2);$$
2. *if* $0 < \beta \leq \bar{\beta}$*, where* $\bar{\beta} = \frac{2t}{2d - c_1 - c_2}$*, then*
   (a) *if* $\alpha < \bar{\alpha}$*, where* $\bar{\alpha} = \frac{\beta(d-t-c_1)}{2t(1-\beta)}$*,*
$$p_1^{NN} = \frac{d}{2} + \frac{t}{\beta}\big(\alpha(1-\beta) + \frac{\beta}{2}\big) + \frac{c_1}{2}, \quad p_2^{NN} = d,$$
$$\pi_1^{NN} = \frac{\beta}{2t}\Big(\frac{d-c_1}{2} + \frac{t}{\beta}\big(\alpha(1-\beta) + \frac{\beta}{2}\big)\Big)^2,$$
$$\pi_2^{NN} = (d - c_2)\Big(1 - \frac{\beta}{2t}\big(\frac{d-c_1}{2} + \frac{t}{\beta}(\alpha(1-\beta) + \frac{\beta}{2})\big)\Big);$$
   (b) *if* $\bar{\alpha} \leq \alpha \leq \tilde{\alpha}$*, where* $\tilde{\alpha} = 1 - \frac{\beta(d-t-c_2)}{2t(1-\beta)}$*,*
$$p_1^{NN} = p_2^{NN} = d,$$
$$\pi_1^{NN} = \big(\alpha(1-\beta) + \frac{\beta}{2}\big)(d - c_1), \quad \pi_2^{NN} = \big((1-\alpha)(1-\beta) + \frac{\beta}{2}\big)(d - c_2);$$
   (c) *if* $\alpha > \tilde{\alpha}$
$$p_1^{NN} = d,\ p_2^{NN} = \frac{d}{2} + \frac{t}{\beta}\big((1-\alpha)(1-\beta) + \frac{\beta}{2}\big) + \frac{c_2}{2},$$
$$\pi_1^{NN} = (d - c_1)\Big(1 - \frac{\beta}{2t}\big(\frac{d-c_2}{2} + \frac{t}{\beta}((1-\alpha)(1-\beta) + \frac{\beta}{2})\big)\Big),$$
$$\pi_2^{NN} = \frac{\beta}{2t}\Big(\frac{d-c_2}{2} + \frac{t}{\beta}\big((1-\alpha)(1-\beta) + \frac{\beta}{2}\big)\Big)^2;$$
3. *if* $\bar{\beta} < \beta < 1$*, then*
   (a) *if* $\alpha < 2\tilde{\alpha} - \bar{\alpha}$*, the results are the same as those when* $\beta \leq \bar{\beta}$ *and* $\alpha < \bar{\alpha}$*;*
   (b) *if* $2\tilde{\alpha} - \bar{\alpha} \leq \alpha \leq 2\bar{\alpha} - \tilde{\alpha}$*,*
$$p_1^{NN} = \frac{2t}{3\beta}\big(1 + \alpha(1-\beta) + \frac{\beta}{2}\big) + \frac{2c_1+c_2}{3},\ p_2^{NN} = \frac{2t}{3\beta}\big(2 - \alpha(1-\beta) - \frac{\beta}{2}\big) + \frac{c_1+2c_2}{3},$$
$$\pi_1^{NN} = \frac{\beta}{2t}\Big(\frac{2t}{3\beta}\big(1 + \alpha(1-\beta) + \frac{\beta}{2}\big) + \frac{c_2-c_1}{3}\Big)^2,$$
$$\pi_2^{NN} = \frac{\beta}{2t}\Big(\frac{2t}{3\beta}\big(2 - \alpha(1-\beta) - \frac{\beta}{2}\big) + \frac{c_1-c_2}{3}\Big)^2;$$
   (c) *if* $\alpha > 2\bar{\alpha} - \tilde{\alpha}$*, the results are the same as those when* $\beta \leq \bar{\beta}$ *and* $\alpha > \tilde{\alpha}$*;*
4. *if* $\beta = 1$*, the results are the same as those when* $\bar{\beta} < \beta < 1$ *and* $2\tilde{\alpha} - \bar{\alpha} \leq \alpha \leq 2\bar{\alpha} - \tilde{\alpha}$*, and can be simplified as,*
$$p_1^{NN} = t + \frac{2c_1+c_2}{3},\ p_2^{NN} = t + \frac{c_1+2c_2}{3},$$
$$\pi_1^{NN} = \frac{1}{2t}(t + \frac{c_2-c_1}{3})^2,\ \pi_2^{NN} = \frac{1}{2t}(t - \frac{c_2-c_1}{3})^2.$$

Here, the threshold values, $\bar{\alpha}$, $\tilde{\alpha}$ and $\bar{\beta}$ are obtained when solving for the optimal solutions in each case, and more details can be found in the proof of Proposition 1 in the Appendix. Note that $\bar{\alpha} = \tilde{\alpha}$ when $\beta = \bar{\beta}$, and $\bar{\alpha} > \tilde{\alpha}$ when $\beta > \bar{\beta}$.

Figure 2 **Solutions to Base Case with $c_1 = 0.1$, $c_2 = 0$, $d = 0.7$ and $t = 0.5$.**



In the following, we use a numerical example ($c_1 = 0.1$, $c_2 = 0$, $d = 0.7$ and $t = 0.5$) to illustrate the results in proposition 1 as shown in Figure 2. The eight cases in Proposition 1 can be graphically categorized into four regions in Figure 2 by using the threshold values, $\bar{\alpha}$, $\tilde{\alpha}$ and $\bar{\beta}$. More specifically, Region I presents case 3(b) and case 4 of Proposition 1; region II shows case 2(a) and case 3(a); region III illustrates case 1 and case 2(b); and region IV is for case 2(c) and case 3(c). As shown in Figure 2, the market structure, i.e., the size of the exclusive offline market α and the competitive online market β, significantly affects retailers' prices and profits.

In case 1 of Proposition 1, when $\beta = 0$, there is no online market and the two retailers sell to their own exclusive offline markets only, the retail prices are thus $p_1^{NN} = p_2^{NN} = d$. Namely, the two retailers charge a retail price equal to an offline consumer's valuation to extract the maximum profit. Consequently, the profits of retailers 1 and 2 can be calculated as are given in case 1 of proposition 1.

In case 2, when the competitive online market exists and its size is small ($\beta < \bar{\beta}$), if α is small as in case 2(a), retailer 1's offline market (i.e., $\alpha(1-\beta)$) is relatively small while retailer 2's offline market (i.e., $(1-\alpha)(1-\beta)$) is relatively large, thus retailer 2 focus more on its offline market and charge a retail price equal to d, while retailer 1 strives to maximize profit from both the offline and online market. Similarly, if α is large as in case 2(c), retailer 1 now has a larger offline market, therefore, charging a retail price d to extract the maximum profit from the offline market, while retailer 2 has to consider both online and offline market when deciding its retail price. In other words, the uneven distribution of the offline market, makes the retailer with a relatively larger offline market focus more on the offline market and charge a retail price equal to d, while the other retailer with a relatively smaller offline market strives to maximize profit from both the offline and online market. When α is medium as in case 2(b), namely, the two retailers have their offline market with the similar size, both retailers focus on their own offline markets, resulting in the same high retail prices d, as in the case when $\beta = 0$.

In case 3, when the size of the competitive online market is large ($\beta > \bar{\beta}$), the uneven distribution of the offline market, i.e., either small α in case 3(a) or large α in case 3(c), makes the retailer with a relatively large offline market focus on the offline market, and the other retailer with a relatively small offline market will strive to maximize profit from both the offline and online markets. However, different from case 2, in case 3(b), when α is medium, because the size of the online market is large, instead of focusing on its own offline market, both retailers are willing to compete in the online market.

In case 4, when $\beta = 1$, i.e., there is no offline market and the two retailers compete only in the online market, the equilibrium retailer prices are $p_1^{NN} = t + \frac{2c_1+c_2}{3}$ and $p_2^{NN} = t + \frac{c_1+2c_2}{3}$. The retailer with higher cost charges a higher price.

Looking at the impact of α and β on the retailers' optimal prices and profits, we find that while the impact of the offline market size α on the prices and profits are always monotonic in all different cases in Proposition 1, i.e., retailer 1 (2)'s profit and price are always increasing (decreasing) in α, the impact of the online market size β on the retailers' profits and prices are not monotonic. In summary, in the base case NN without PM or QI, the size of the online and offline market significantly affects retailers' pricing strategy and profits. We next study how the focal retailer 1 deliberates over quality improvement strategy and price-matching strategy, and how different strategies can impact the retailers' optimal prices and profits.

4.2. Price-Matching

In reality, many retailers indeed offer price-matching policy in competition, such as Target and Bestbuy. In our model, when retailer 1 chooses to compete with retailer 2 via price-matching instead of quality improvement, in order to find out the equilibrium outcome, we need to compare four cases, NN, MN, NM, and MM. Similar to the base case NN, we next solve the cases with symmetric price-matching (MM) and asymmetric price-matching (MN and NM). In each case, the retailers simultaneously decide on their retail prices, after observing each other's strategies.

In case MM, because the two retailers both adopt price-matching, consumers can buy the product from any retailer and apply for price-matching if needed, and thus the final price that the consumers pay is the same, regardless of which retailer they buy from. Therefore, if a consumer located at x ($0 \leq x \leq 1$) in the online market buys from retailer 1 or retailer 2, the utility is $u_1 = 1 - \min(p_1^{MM}, p_2^{MM}) - tx$ and $u_2 = 1 - \min(p_1^{MM}, p_2^{MM}) - t(1-x)$, respectively. As a result, the retailers share the online market evenly, and the profits can be written as,

$$\max_{p_1^{MM}} \pi_1^{MM} = \big(\alpha(1-\beta) + \frac{\beta}{2}\big)\big(\min(p_1^{MM}, p_2^{MM}) - c_1\big) \tag{3}$$

$$\max_{p_2^{MM}} \pi_2^{MM} = \big((1-\alpha)(1-\beta) + \frac{\beta}{2}\big)\big(\min(p_1^{MM}, p_2^{MM}) - c_2\big). \tag{4}$$

In case MN, retailer 1 adopts price-matching policy but retailer 2 does not. Therefore, if an online consumer located at x ($0 \leq x \leq 1$) buys from retailer 1, her utility is $u_1 = 1 - \min(p_1^{MN}, p_2^{MN}) - tx$, and if she buys from retailer 2, her utility is $u_2 = 1 - p_2^{MN} - t(1-x)$. Namely, for the consumers, retailer 1's effective retail price is $\min(p_1^{MN}, p_2^{MN})$. By letting $u_1 = u_2$ and solving for x, a consumer located at $\hat{x} = \frac{p_2^{MN} - \min(p_1^{MN}, p_2^{MN}) + t}{2t}$ is indifferent between buying from retailer 1 and retailer 2. As a result, online consumers with $x \leq \hat{x}$ will buy from retailer 1, and those with $x > \hat{x}$ will buy from retailer 2. Consequently, the two retailers' profits can be written as follows:

$$\max_{p_1^{MN}} \pi_1^{MN} = \big(\alpha(1-\beta) + \beta\frac{p_2^{MN} - \min(p_1^{MN}, p_2^{MN}) + t}{2t}\big)\big(\min(p_1^{MN}, p_2^{MN}) - c_1\big), \tag{5}$$

$$\max_{p_2^{MN}} \pi_2^{MN} = \big((1-\alpha)(1-\beta) + \beta\frac{\min(p_1^{MN}, p_2^{MN}) - p_2^{MN} + t}{2t}\big)(p_2^{MN} - c_2). \tag{6}$$

Similarly, in case NM, the two retailers' profits can be written as follows:

$$\max_{p_1^{NM}} \pi_1^{NM} = \big(\alpha(1-\beta) + \beta\frac{\min(p_1^{NM}, p_2^{NM}) - p_1^{NM} + t}{2t}\big)\big(p_1^{NM} - c_1\big), \tag{7}$$

$$\max_{p_2^{NM}} \pi_2^{NM} = \big((1-\alpha)(1-\beta) + \beta\frac{p_1^{NM} - \min(p_1^{NM}, p_2^{NM}) + t}{2t}\big)(\min(p_1^{NM}, p_2^{NM}) - c_2). \tag{8}$$

Solving cases MN, NM, and MM, we can find out the corresponding optimal prices and profit for the two retailers (The results are given in the Appendix.). These results allow us to investigate retailers' equilibrium decisions regarding whether to offer price-matching in a normal form game. Assuming that if a retailer is indifferent between choosing or not choosing price-matching, the retailer will not choose price-matching,[3] and the next proposition characterizes the pure strategy Nash equilibrium for retailers' price-matching decisions.

PROPOSITION 2. *The equilibrium price-matching outcome E is:*

(i) $E=MM$ *when* $\bar{\beta}<\beta<1$ *and* $\tilde{\alpha}<\alpha<\bar{\alpha}$, *or* $\beta=1$;

(ii) $E=MN$ *when* $\bar{\beta}<\beta<1$ *and* $\alpha\geq\bar{\alpha}$, *or* $\beta\leq\bar{\beta}$ *and* $\alpha>\tilde{\alpha}$;

(iii) $E=NM$ *when* $\bar{\beta}<\beta<1$ *and* $\alpha\leq\tilde{\alpha}$, *or* $\beta\leq\bar{\beta}$ *and* $\alpha<\bar{\alpha}$;

(iv) $E=NN$ *when* $\beta\leq\bar{\beta}$ *and* $\bar{\alpha}\leq\alpha\leq\tilde{\alpha}$, *or* $\beta=0$.

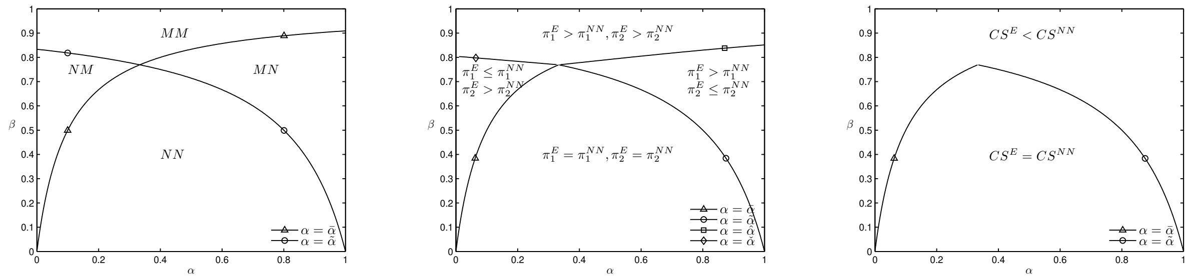
Figure 3(a) visualizes the results from Proposition 2 and demonstrates how the equilibrium outcome changes with the market structure, i.e., α and β. Proposition 2 and Figure 3(a) together show that, first, when the size of the competitive online market is large, one or both retailers are willing to adopt price-matching, and when the size of the competitive online market is small, none of them will offer price-matching. Second, the asymmetric price-matching strategy is due to the asymmetric share of the offline market. More specifically, retailer 1 (2) would adopt PM policy if $\alpha>\tilde{\alpha}$ ($\alpha<\bar{\alpha}$) in the equilibrium. In other words, when α is large or small, namely, one retailer takes a relatively large share of the offline market, this retailer is willing to offer price-matching in equilibrium, while the other retailer with a relatively small share of the offline market does not offer price-matching. It is also interesting to note that $\tilde{\alpha}$ ($\bar{\alpha}$) increases (decreases) in c_2 (c_1), and is independent of c_1 (c_2). Namely, the region for one retailer to adopt PM is not impacted by his own cost, but shrinks as his rival's cost increases. This finding suggests that when deciding whether adopting the PM policy, a retailer should pay more attention to the rival's cost. In addition, if we take into account the fixed cost of offering price-matching policy, the PM adoption region in the equilibrium would shrink.

The next corollary shows how retailers' prices and profits, and consumer surplus change in equilibrium, compared with the base case.

COROLLARY 1. *Compared with the base case NN, in the equilibrium E when one or both retailers adopt price-matching,*

[3] This assumption helps us clearly illustrate the equilibrium in the normal form game, and is also consistent with the practice that a retailer may incur an extra cost when offering the price-matching policy.

Figure 3 Equilibrium decisions on price-matching, retail profits and consumer surplus.



(a) Equilibrium outcomes (b) Retail profits (c) Consumer surplus

Note. $c_1 = 0.1$, $c_2 = 0$, $d = 0.7$ and $t = 0.5$. $\check{\alpha}$ and $\hat{\alpha}$ are given in the proof of Corollary 1.

(i) the two retailer's prices are higher;

(ii) the two retailers' profits are higher in symmetric PM case; the retailer who adopts price-matching always earns more profits, while the retailer without price-matching can be hurt in asymmetric PM cases when its offline market share is low;

(iii) the consumer surplus is lower.

Corollary 1 reveals several interesting insights. First, price-matching can increase the retail prices in competition, leading to price collusion in dual-channel retailing. It is because when one retailer offers price-matching policy, the competitor's price will be matched if it is lower, therefore retailers have the incentive to set higher prices under price-matching. In other words, the degree of the competition between the two retailers is weakened by price-matching policy. In fact, we find that the equilibrium retail prices are always $p_1^E = p_2^E = d$, the highest that the retailers can charge to maintain both offline and online markets. This is consistent with existing literature that the price-matching policy is anti-competition (Coughlan & Shaffer 2009; Nalca et al. 2013). Second, price-matching cannot always improve the profits of the two retailers. When both retailers offer PM in the equilibrium, both retailers can earn more profits than the base case. This is because price-matching policy significantly weakens the competition, and their retail prices are increased greatly. However, when only one retailer offers PM, the profit of the retailer that does not adopt price-matching in equilibrium can be higher or lower than the base case. In particular, the profit of the retailer without price-matching is lower if its share of offline market is small, which is mainly because that the retailer can only earn a half share of online market when its rival offers price-matching and its total market share is small due to its small offline market. This indicates that under dual-channel retailing, it is not always beneficial for both retailers to adopt price-matching and that the market structure (i.e., α and β) can play a vital role in determining the price-matching equilibrium and retailers' profits. We can also find that the profit of a retailer increases with its share of offline market, but increases with the online market size only when its share of offline market is small, since so does its demand. Third, because price-matching leads to price collusion and the retailers charge higher retail prices, the consumer surplus is reduced if any retailer adopts the policy in the equilibrium. Our results also show that the consumer surplus in the equilibrium is independent of α and increases in β. This is because the offline consumers have zero surplus and the online consumers have positive surplus, when $p_1^E = p_2^E = d$.

The results in Proposition 2 and Corollary 1 can explain why some dual-channel retailers often offer price-matching policy during online sales events. Because online sellers advertise heavily for the sales events, the online consumer ratio would increase temporarily, and the dual-channel retailer with large offline market share can benefit from price-matching. If the online consumer ratio can increase to a very high level, the two retailers can both be better off by offering the policy. It is observed that, Suning, a dual-channel retailer with large offline market in China, often announced the policy during online sales events in recent years, for example, the mid-year sales event in 2020. In addition, Gome, another dual-channel retailer in China, also often offered the policy during such sales events in the past years.

Figures 3(b) and 3(c) illustrate the regions when price-matching can benefit or hurt retailers and consumers in the equilibrium compared with the base case. Our results show that when the size of the online market is not too large, price-matching can benefit the retailer with a relatively larger share of the offline market and hurt the other retailer with a relatively small share of the offline market. For retailer 1, it is clear that the PM policy is beneficial only when α or β is high, as shown in Figure 3(b). Because the policy can always improve the retail prices, the consumer surplus is reduced if any retailer adopts the PM policy in the equilibrium, and remain the same if no retailer adopts the policy, as shown in Figure 3(c).

4.3. Quality Improvement

Although many retailers still offer price-matching policy, a number of retailers have withdrawn the policy, and some of them turn to quality improvement strategy and offer products with higher quality. For example, Waitrose in UK chose to improve its product quality instead of offering price-matching in 2018. In this section, we take into account the focal retailer 1's decision on QI strategy, and derive retailer 1's optimal quality level, and the equilibrium prices and profits of the two retailers.

In this case, retailer 1 decides on the optimal quality improvement level q and its retail price p_1^{QI}, while retailer 2 decides on its retail price p_2^{QI} only. Because QI brings extra cost to the retailer and gives additional valuation to the consumers, we use $c(q)$ to denote the cost of quality improvement q for retailer 1, and $u(q)$ to represent the additional utility that consumers can obtain when they buy the product of retailer 1. Without loss of generality, we assume that $c(0) = u(0) = 0$ and $c'(0) < u'(0)$, otherwise retailer 1 would never choose to improve quality. We also assume that the cost and the marginal cost increase in quality improvement level, i.e., $c'(q) > 0$ and $c''(q) \geq 0$, and the utility increases and the marginal utility decreases in quality improvement, i.e., $u'(q) > 0$ and $u''(q) \leq 0$. These assumptions are commonly adopted in the literature (e.g., Chambers et al. 2006, Jing 2017, Keskin & Birge 2019 and Rao & Turut 2019).

Given the retailers' prices p_1^{QI} and p_2^{QI}, while consumers in the offline market will still buy from the corresponding retailer, consumers in the online market make purchase decisions by maximizing their utility. If an online consumer located at x buys from retailer 1, her utility is $u_1 = 1 - p_1^{QI} + u(q) - tx$, and if she buys from retailer 2, her utility is $u_2 = 1 - p_2^{QI} - t(1-x)$. By letting $u_1 = u_2$ and solving for x, a consumer located at $\hat{x} = \frac{p_2^{QI} - p_1^{QI} + u(q) + t}{2t}$ is indifferent between buying from retailer 1 and retailer 2. Therefore, online consumers with $x \leq \hat{x}$ will buy from retailer 1, and those with $x > \hat{x}$ will buy from retailer 2. Consequently, the two retailers' profit functions are as follows,

$$\max_{p_1^{QI}, q} \pi_1^{QI} = \big(\alpha(1-\beta) + \beta \frac{p_2^{QI} - p_1^{QI} + u(q) + t}{2t}\big)(p_1^{QI} - c_1 - c(q)), \tag{9}$$

$$\max_{p_2^{QI}} \pi_2^{QI} = \big((1-\alpha)(1-\beta) + \beta \frac{p_1^{QI} - p_2^{QI} - u(q) + t}{2t}\big)(p_2^{QI} - c_2). \tag{10}$$

Solving (9) and (10), we get the optimal QI decision, the equilibrium prices and resulting profits, which are summarized in the following proposition.

Proposition 3. *Retailer 1's optimal quality improvement level* q^* *can be uniquely solved from* $c'(q^*) = u'(q^*)$. *The equilibrium prices are* $p_1^{QI} = p_1^{NN}(c_1^{QI}) + u(q^*)$ *and* $p_2^{QI} = p_2^{NN}(c_1^{QI})$, *and the resulting profits are* $\pi_1^{QI} = \pi_1^{NN}(c_1^{QI})$ *and* $\pi_2^{QI} = \pi_2^{NN}(c_1^{QI})$, *where* $c_1^{QI} = c_1 + c(q^*) - u(q^*)$.

Here, $p_1^{NN}(c_1^{QI})$, $p_2^{NN}(c_1^{QI})$, $\pi_1^{NN}(c_1^{QI})$ and $\pi_2^{NN}(c_1^{QI})$ represent p_1^{NN}, p_2^{NN}, π_1^{NN} and π_2^{NN} when $c_1 = c_1^{QI}$. According to Proposition 3, the optimal quality improvement level q^* is independent of the market structure and the pricing decision, but only determined by $c(q)$ and $u(q)$. Note that $c(q^*) < u(q^*)$, therefore, improving retailer 1's quality by q^* is equivalent to reducing its unit cost by $u(q^*) - c(q^*)$, i.e., from c_1 to c_1^{QI}. Therefore, the equilibrium and profits are similar as those in Lemma 1 by replacing c_1 with c_1^{QI}.

The next corollary compares the retailer's prices and profits, and consumer surplus in this case with those in the base case.

Corollary 2. *Compared with the base case NN, when retailer 1 adopts QI strategy,*

(i) retailer 1's price is higher, and retailer 2's price is lower;

(ii) retailer 1's profit is higher, and retailer 2's profit is lower;

(iii) the consumer surplus is higher.

Corollary 2 reports several key findings. First, compared with the base case NN, retailer 1 increases its retail price when improving its quality level, while retailer 2 reduces its price. This is because retailer 1 need to cover the cost associated with quality improvement, but the utility of retailer 1's product is higher. Consequently, retailer 2 has to reduce its price in the competition. Second, improving quality is equivalent to reducing the selling cost, retailer 1 can obtain competitive advantage and thus earn more profits, therefore, retailer 2's profit decreases. Third, although retailer 1's price is higher, the effective price the consumers pay when purchasing from retailer 1 is $p_1^{NN}(c_1^{QI})$ (lower than p_1^{NN}), and retailer 2's price is lower, which results in higher consumer surplus. The findings suggest that, QI strategy could be an alternative to PM strategy to help retailers earn more profits in retail competition.

4.4. Optimal Strategy

In this section, we will compare QI strategy and PM strategy, and examine when one strategy outperforms the other. Based on the results in §4.2 and §4.3, the following proposition characterizes when a retailer should adopt QI strategy or PM strategy.

Proposition 4. *There always exists a threshold value $\beta^{QI}(\alpha)$ such that when $\beta \leq \beta^{QI}(\alpha)$, retailer 1 chooses QI strategy.*

Due to the complexity of the form of $\beta^{QI}(\alpha)$, we discuss it in the proof of Proposition 4 in the Appendix. Proposition 4 shows that QI strategy outperforms PM strategy, only when the online market is relatively small, i.e., $\beta \leq \beta^{QI}(\alpha)$. The reason is that, when the online market is small, the retailers focus more on their exclusive offline markets, and their retail prices are already very high as is shown in the base case. Therefore, although price-matching can further increase the retail price and lead to price collusion, the increase in the profits is not large. In this case, quality improvement performs better than price-matching. In contrast, when $\beta > \beta^{QI}(\alpha)$, it is better to adopt price-matching instead of quality improvement.

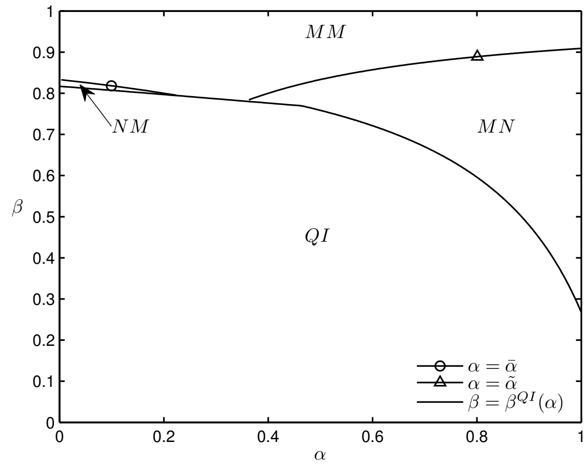
In addition, because we have shown that $\pi_1^{QI} \geq \pi_1^{NN}$ in Corollary 2, it is easy to see $\max(\pi_1^{QI}, \pi_1^{E}) \geq \pi_1^{NN}$, i.e., retailer 1 can always improve its profit by choosing between PM and QI, which indicates that NN will never be the final equilibrium with the choice of QI. However, our results indicate that, when the retailers engage in PM competition, the final equilibrium could be MN, NM and MM. In other words, when retailer 1 choose not to improve quality, it may be retailer 2 who will adopt price-matching, rather than retailer 1, in the equilibrium. Also, since we have $\pi_1^{E} \leq \pi_1^{NN}$ when α is low shown in Corollary 1 , retailer 1 must choose QI when its offline market share is low. Moreover, according to Proposition 4 and Corollaries 1 and 2, we can see that retailer 2 can earn more profits only when retailer 1 does not adopt QI strategy, and the consumer surplus is higher only when retailer 1 adopts QI strategy.

The findings are consistent with the practice. It is observed that, some retailers such as Waitrose, adopted QI strategy after offering PM policy for several years. That is because QI strategy can help the retailer earn more profits when its offline market share is not high enough. Nevertheless, the retailer may need a long-time investment to acquire the ability to improve the product quality. However, QI strategy can help the retailer maintain competitive advantage for a prolonged time, because the strategy, unlike PM, cannot be easily followed by the competitor in the short term.

Figure 4 visualizes the optimal strategy in equilibrium. We can see that $\beta^{QI}(\alpha)$ decreases in α. QI strategy outperforms PM strategy for retailer 1 when the online market is small or its share of offline market is low, i.e., in the region below the curve $\beta = \beta^{QI}(\alpha)$. When the online market is big, PM strategy performs better, i.e., in the region above the line $\beta = \beta^{QI}(\alpha)$, and we can see that the equilibrium outcome on PM policy could be MN, NM and MM .

We finally examine the impact of market structure, i.e., α and β, on the retailers' prices and profits, and consumer surplus.

Figure 4 **The optimal strategy.**



Note. $c_1 = 0.1$, $c_2 = 0$, $d = 0.7$, $t = 0.5$, $c(q) = 1.5q^2$, and $u(q) = 0.1q^{1/2}$.

Corollary 3. *Under the optimal strategy,*

(i) the two retailers' prices always increase in their share of offline market, and decrease in β;

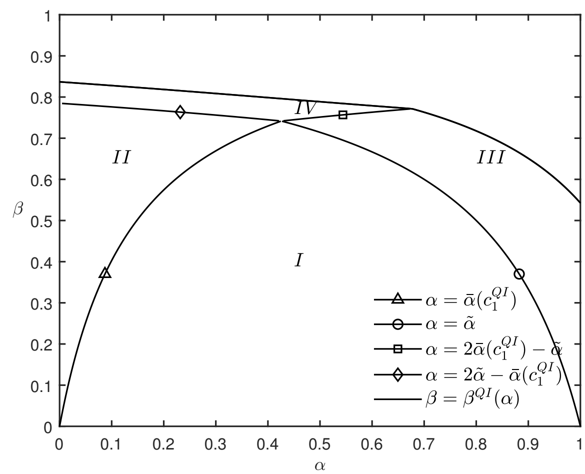
(ii) the two retailers' profits always increase in their share of offline market, and can either increase or decrease in β;

(iii) the consumer surplus can either increase or decrease in α, and always increase in β.

Because we have discussed the impact of market structure on the equilibrium prices, profits and consumer surplus when the retailers engage in PM game in Section 4.2, here we focus on examining the results when the optimal strategy is QI, i.e., when $\beta \leq \beta^{QI}(\alpha)$. Observing the retailers' prices, we can find that p_1^{QI} increases in α and decreases in β, and p_2^{QI} decreases in α and decreases in β. Namely, the two retailers' prices increase as their share of offline market increase, and decrease as the online market size increases. This is because when one retailer's offline market becomes larger, this retailer is more willing to focus only on its own offline market, leading the retail price to reach the offline consumers' valuation of its product. Moreover, a larger online market encourages the two retailers to compete, thus resulting in lower retail prices from both retailers.

For the retailers' profits, we find that π_1^{QI} (π_2^{QI}) always increases (decreases) in α, because one retailer has a bigger market share as its offline market size increases. Also, π_1^{QI} (π_2^{QI}) first increases and then decreases in β when $\alpha \leq 1/2$ ($\alpha \geq 1/2$), and always decreases in β when $\alpha > 1/2$ ($\alpha < 1/2$). That means a higher online competitive market only benefits a retailer when its share of offline market is small. This indicates that competition can benefit the retailer with a smaller offline market. The intuition is that when the competitive online market becomes larger, it gives the retailer with a small offline market the opportunity to access a larger market, even though such a retailer has to face competition. However, when the level of competition becomes too high, this benefit will diminish.

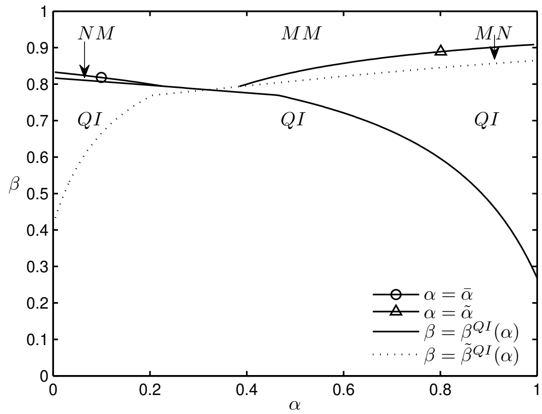
Figure 5 Regions under the optimal strategy QI.



Note. $c_1 = 0.1$, $c_2 = 0$, $d = 0.7$, $t = 0.5$, $c(q) = 1.5q^2$, and $u(q) = 0.2q^{1/2}$.

The consumer surplus is determined by retailers' prices and consumer structure. We find that it always increases in β, because we have shown that the retailers' prices decrease as β increases. However, the impact of α on consumer surplus is complicated. We next explain the impact of α using the four regions under QI strategy in Figure 5, where the equilibrium prices are different and the consumer surplus changes differently as α increases. When $\bar{\alpha}(c_1^{QI}) \le \alpha \le \tilde{\alpha}$ (region I), the two retailers' prices are always equal to the valuation of offline consumers, so the consumer surplus is independent of α. When $\alpha < \min(\bar{\alpha}(c_1^{QI}), 2\tilde{\alpha} - \bar{\alpha}(c_1^{QI}))$ (region II), we can find that the consumer surplus decreases in α, which is mainly because retailer 1's price increases in α and retailer 2's price is always d. We can similarly find that the consumer surplus increases in α when $\alpha > \max(\tilde{\alpha}, 2\bar{\alpha}(c_1^{QI}) - \tilde{\alpha})$ (region III). When $2\tilde{\alpha} - \bar{\alpha}(c_1^{QI}) \le \alpha \le 2\bar{\alpha}(c_1^{QI}) - \tilde{\alpha}$ (region IV), our results show that $p_1^{QI} + p_2^{QI}$ is independent of α and p_1^{QI} increases in α. Also, the surplus of offline consumers increases in α when retailer 1's share of offline market is less than a half, i.e., $\alpha \le 1/2$, and decreases in

Figure 6 **Equilibrium strategy on QI and PM.**



Note. $c_1 = 0.1$, $c_2 = 0$, $d = 0.7$, $t = 0.5$, $c_1(q) = 1.5q^2$, $c_2(q) = 1.6q^2$, and $u(q) = 0.2q^{1/2}$. QI means the two retailers both implement QI strategy.

α when $\alpha \geq 1/2$, while the surplus of online consumers does not change with α, and thus the total consumer surplus firstly increases and then decreases in α.

In the previous analyses, only the focal retailer can choose QI. If retailer 2 can also implement QI, it can be shown that, according to Proposition 4, if retailer 1 does not implement QI strategy, then there exists a threshold value $\tilde{\beta}^{QI}(\alpha)$, such that when $\beta < \tilde{\beta}^{QI}(\alpha)$, the optimal strategy for retailer 2 is to choose QI. In addition, we can find that the optimal quality improvement level of two retailers would be the same if they have the same cost function regarding quality improvement level according to Proposition 3. Otherwise, the retailer who has cost advantages in quality improvement can obtain a higher optimal improvement level. Moreover, we know that one retailer would be hurt when the other retailer unilaterally implements QI strategy according to Corollary 2, and deviating to QI strategy under such situation can help the retailer earn more profits. Therefore, the two retailers will both choose QI strategy when $\beta < \beta^{QI}(\alpha)$ or $\beta < \tilde{\beta}^{QI}(\alpha)$. Figure 6 numerically analyzes the retailers' equilibrium strategy on QI and PM. The figure shows that the retailers both implement QI strategy when the ratio of online consumers is low. And when $\beta \geq \beta^{QI}(\alpha)$ and $\beta \geq \tilde{\beta}^{QI}(\alpha)$, the two retailers directly engage in price-matching game and the equilibrium can be obtained according to Proposition 2. In addition, we find that the main insights still hold when both retailers can choose QI. For example, the consumer surplus is higher only when the retailers choose QI strategy, and the retailer with cost advantage in quality improvement can be always better off.

5. Conclusion and Future Research

Managing both online and offline channels is a challenging task when there is competition. In this paper, we focus on the setting of dual channels, in which two retailers compete in an online market while maintaining their own offline markets. Such a setting allows us to investigate the impact of the market structure on a retailer's deliberation over two alternative strategies, quality improvement and price-matching.

Our results show that a retailer can earn more profits by choosing the optimal strategy between price-matching and quality improvement, and quality improvement outperforms price-matching when the competitive market is small. With quality improvement strategy, the retailer can earn more profits, the consumer surplus can also be improved, but the other retailer can be hurt. When the retailers choose price-matching, different market structures, i.e., the size of the retailers' online and offline markets, can lead to different price-matching equilibria. More specifically, neither or both retailers applying price-matching, or only one retailer applies price-matching can all become the final equilibrium outcome. We also find that, although one or both retailers adopting price-matching can lead to higher retail prices, resulting in price collusion in the market, such price collusion reduces the consumer surplus, and does not always benefit both retailers. In order to benefit from price-matching, a retailer needs to have a sufficiently large offline market. Therefore, this study highlights the importance of managing the interaction of a retailer's offline and online market, when retailers make quality improvement and price-matching decisions.

The simplicity of our model necessarily involves some limitations that can lead to fruitful future research. First, we assume that the two retailers apply the same price across different channels and offer price-matching for all consumers. Further research can consider differentiated pricing or price-matching policies for a retailer's online and offline market. Second, all consumers in our model are assumed to be informed about the two retailers' prices and policies before purchasing, and they have no cost to apply for price-matching after purchasing. Future research can study the case when consumers may incur different costs to apply for price-matching in the dual-channel setting. In this case, price-matching policy can be used as a tool to differentiate consumers. Third, we assume that the total demand is deterministic in the paper, and future research can also examine the problem under demand uncertainty. Under such settings, the consumers' purchasing behavior when stock-out occurs should be modeled, and the optimal inventory level of the retailers should be determined, so that the model would be more complicated and the results would be more fruitful. Last but not least, our paper only studies the focal retailer's QI decision. It will be interesting to study the case when both retailers may deliberate over QI and PM strategies in the dual-channel setting. We hope our research could encourage more studies in these directions.

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Appendix

Proof of Proposition 1. In this proof, we first show that there are no equilibrium prices for $p_1^{NN} > d$ or $p_2^{NN} > d$ in the game. We then derive the optimal equilibrium prices by solving the first-order conditions of the profit functions 1 and 2, determine the conditions regarding α and β under which the derived equilibrium prices satisfy $p_1^{NN} \le d$ and $p_2^{NN} \le d$, and check the corner solutions ($p_1^{NN} = d$ or $p_2^{NN} = d$) and their conditions.

Non-existence of equilibrium prices for $p_1^{NN} > d$ or $p_2^{NN} > d$ is shown as follows. When $|p_1^{NN} - p_2^{NN}| > t$, the profit of the retailer with higher price (more than d) is 0 and there does not exist an equilibrium. When $|p_1^{NN} - p_2^{NN}| \le t$, the following three scenarios show that there also does not exist an equilibrium. (1) If $p_1^{NN} > d$ and $p_2^{NN} > d$, $\pi_1^{NN} = \beta \frac{p_2^{NN} - p_1^{NN} + t}{2t}(p_1^{NN} - c_1)$ and $\pi_2^{NN} = \beta \frac{p_1^{NN} - p_2^{NN} + t}{2t}(p_2^{NN} - c_2)$. The optimal equilibrium prices are $p_1^{NN} = t + \frac{2c_1 + c_2}{3}$ and $p_2^{NN} = t + \frac{c_1 + 2c_2}{3}$, which are not consistent with the assumption $d \ge t + \max(c_1, c_2)$. (2) If $p_1^{NN} > d$ and $p_2^{NN} \le d$, $\pi_1^{NN} = \beta \frac{p_2^{NN} - p_1^{NN} + t}{2t}(p_1^{NN} - c_1)$ and $\pi_2^{NN} = ((1-\alpha)(1-\beta) + \beta \frac{p_1^{NN} - p_2^{NN} + t}{2t})(p_2^{NN} - c_2)$. The optimal equilibrium prices are $p_1^{NN} = \frac{p_2^{NN} + t + c_1}{2}$ and $p_2^{NN} = \frac{p_1^{NN} + t + c_2}{2} + \frac{t(1-\alpha)(1-\beta)}{\beta}$, which are not in accordance with $p_1^{NN} > d$ and $p_2^{NN} \le d$. (3) If $p_1^{NN} \le d$ and $p_2^{NN} > d$, we can similarly show that there does not exist an equilibrium.

The optimal equilibrium prices for $p_1^{NN} \le d$ and $p_2^{NN} \le d$ are derived as follows. It is easy to see that there does not exist an equilibrium for $|p_1^{NN} - p_2^{NN}| > t$, as the retailer with a lower price can gain a profit improvement by improving its price. Hence, we just need to look for equilibrium prices with $p_1^{NN} \le d$, $p_2^{NN} \le d$, and $|p_1^{NN} - p_2^{NN}| \le t$. Solving the first-order conditions of the profit functions 1 and 2 yields the following equilibrium prices,

$$p_1^{NN} = \frac{2t}{3\beta}(1 + \alpha(1-\beta) + \frac{\beta}{2}) + \frac{2c_1 + c_2}{3} \text{ and } p_2^{NN} = \frac{2t}{3\beta}(2 - \alpha(1-\beta) - \frac{\beta}{2}) + \frac{c_1 + 2c_2}{3}.$$

We next check under what conditions the prices above satisfy $p_1^{NN} \le d$, $p_2^{NN} \le d$, and $|p_1^{NN} - p_2^{NN}| \le t$. When $\beta = 1$, we can see $p_1^{NN} \le d$ and $p_2^{NN} \le d$ with $d \ge t + \max(c_1, c_2)$, and when $\beta < 1$, we need $2 - \frac{3\beta d}{2t} + \frac{3\beta(c_1 + c_2)}{4t} \le \alpha(1-\beta) + \frac{\beta}{2} + \frac{\beta(c_1 - c_2)}{4t} \le \frac{3\beta d}{2t} - \frac{3\beta(c_1 + c_2)}{4t} - 1$, i.e., $2\tilde{\alpha} - \bar{\alpha} \le \alpha \le 2\bar{\alpha} - \tilde{\alpha}$ (which implies $\beta > \bar{\beta}$). In addition, with $d \le \frac{t}{\beta} + \frac{t}{2} + \frac{c_1 + c_2}{2}$, we can see that $\frac{3\beta d}{2t} - \frac{3\beta(c_1 + c_2)}{4t} - 1 < \frac{1}{2} + \frac{3\beta}{4}$, which ensures $\frac{1}{2} - \frac{3\beta}{4} \le \alpha(1-\beta) + \frac{\beta}{2} +$

$\frac{\beta(c_1-c_2)}{4t} \le \frac{1}{2}+\frac{3\beta}{4}$, i.e., $|p_1^{NN}-p_2^{NN}| \le t$. Therefore, under the condition $t+\max(c_1,c_2) \le d \le \frac{t}{\beta}+\frac{t}{2}+\frac{c_1+c_2}{2}$, the equilibrium prices shown above fulfill $p_1^{NN} \le d$, $p_2^{NN} \le d$ and $|p_1^{NN}-p_2^{NN}| \le t$, when $\beta=1$ (**Case 4**), or when $\bar{\beta}<\beta<1$ and $2\tilde{\alpha}-\bar{\alpha} \le \alpha \le 2\bar{\alpha}-\tilde{\alpha}$ (**Case 3(b)**). Please note that an interior solution ($p_1^{NN}<d$ and $p_2^{NN}<d$) exists only in such situations. Otherwise, i.e., when $\beta \le \bar{\beta}$, or $\bar{\beta}<\beta<1$ and $\alpha>2\bar{\alpha}-\tilde{\alpha}$, or $\bar{\beta}<\beta<1$ and $\alpha<2\tilde{\alpha}-\bar{\alpha}$, there are three possible corner solutions: (1) $p_1^{NN}=d$ and $p_2^{NN}=\frac{d}{2}+\frac{t}{\beta}((1-\alpha)(1-\beta)+\frac{\beta}{2})+\frac{c_2}{2}$ ($p_2^{NN}<d$ when $\alpha>\tilde{\alpha}$), which is derived by $\frac{\partial \pi_2^{NN}}{\partial p_2^{NN}}=(1-\alpha)(1-\beta)+\beta\frac{p_1^{NN}-2p_2^{NN}+t+c_2}{2t}=0$ given $p_1^{NN}=d$; (2) $p_2^{NN}=d$ and $p_1^{NN}=\frac{d}{2}+\frac{t}{\beta}(\alpha(1-\beta)+\frac{\beta}{2})+\frac{c_1}{2}$ ($p_1^{NN}<d$ when $\alpha<\bar{\alpha}$), which is derived by $\frac{\partial \pi_1^{NN}}{\partial p_1^{NN}}=0$ given $p_2^{NN}=d$; and (3) $p_1^{NN}=p_2^{NN}=d$.

It can be verified that, when $\bar{\beta}<\beta<1$ and $\alpha>2\bar{\alpha}-\tilde{\alpha}$ (**Case 3(c)**), or when $\beta \le \bar{\beta}$ and $\alpha>\tilde{\alpha}$ (**Case 2(c)**), the equilibrium prices are,

$$p_1^{NN}=d \text{ and } p_2^{NN}=\frac{d}{2}+\frac{t}{\beta}((1-\alpha)(1-\beta)+\frac{\beta}{2})+\frac{c_2}{2}.$$

Given $p_1^{NN}=d$, $\frac{\partial \pi_2^{NN}}{\partial p_2^{NN}}=0$ yields $p_2^{NN}=\frac{d}{2}+\frac{t}{\beta}((1-\alpha)(1-\beta)+\frac{\beta}{2})+\frac{c_2}{2}$, which is optimal for retailer 2. Given $p_2^{NN}=\frac{d}{2}+\frac{t}{\beta}((1-\alpha)(1-\beta)+\frac{\beta}{2})+\frac{c_2}{2}$, $\frac{\partial \pi_1^{NN}}{\partial p_1^{NN}}=\alpha(1-\beta)+\beta\frac{p_2^{NN}-2p_1^{NN}+t+c_1}{2t}>0$ holds for $p_1^{NN} \le d$, when $\bar{\beta}<\beta<1$ and $\alpha>2\bar{\alpha}-\tilde{\alpha}$ or when $\beta \le \bar{\beta}$ and $\alpha>\tilde{\alpha}$. This is to say that retailer 1 would not set its price lower than d. In addition, retailer 1 would not set its price higher than d because $\frac{\partial \pi_1^{NN}}{\partial p_1^{NN}}=\beta\frac{p_2^{NN}-2p_1^{NN}+t+c_1}{2t}<0$ holds for $p_1^{NN} \ge d$ given $p_2^{NN}=\frac{d}{2}+\frac{t}{\beta}((1-\alpha)(1-\beta)+\frac{\beta}{2})+\frac{c_2}{2}<d$ and $d \ge t+\max(c_1,c_2)$. Similarly, when $\bar{\beta}<\beta<1$ and $\alpha<2\tilde{\alpha}-\bar{\alpha}$ (**Case 3(a)**), or when $\beta \le \bar{\beta}$ and $\alpha<\bar{\alpha}$ (**Case 2(a)**), we can show that

$$p_1^{NN}=\frac{d}{2}+\frac{t}{\beta}(\alpha(1-\beta)+\frac{\beta}{2})+\frac{c_1}{2} \text{ and } p_2^{NN}=d$$

are the equilibrium prices. In addition, when $\beta \le \bar{\beta}$ and $\bar{\alpha} \le \alpha \le \tilde{\alpha}$ (**Case 2(b)**), it can be shown that the equilibrium prices are,

$$p_1^{NN}=d \text{ and } p_2^{NN}=d.$$

Given $p_1^{NN}=p_2^{NN}=d$, the two retailers would not reduce their prices because $\frac{\partial \pi_1^{NN}}{\partial p_1^{NN}}=\alpha(1-\beta)+\beta\frac{p_2^{NN}-2p_1^{NN}+t+c_1}{2t} \ge 0$ holds for $p_1^{NN} \le d$ with $\alpha \ge \bar{\alpha}$ and $\frac{\partial \pi_2^{NN}}{\partial p_2^{NN}}=(1-\alpha)(1-\beta)+\beta\frac{p_1^{NN}-2p_2^{NN}+t+c_2}{2t} \ge 0$ holds for $p_2^{NN} \le d$ with $\alpha \le \tilde{\alpha}$, and also would not improve their prices because $\frac{\partial \pi_1^{NN}}{\partial p_1^{NN}}=\beta\frac{p_2^{NN}-2p_1^{NN}+t+c_1}{2t} \le 0$ for $p_1^{NN}>d$ and $\frac{\partial \pi_2^{NN}}{\partial p_2^{NN}}=\beta\frac{p_1^{NN}-2p_2^{NN}+t+c_2}{2t} \le 0$ for $p_2^{NN}>d$. The equilibrium prices $p_1^{NN}=p_2^{NN}=d$ also apply when $\beta=0$ in particular (**Case 1**). Please note that we need $d \le 1-\frac{t}{2}$ to ensure that the online market can always be covered by the two retailers. To explore the impact of β from 0 to 1 on the equilibrium results, we consider $t+\max(c_1,c_2) \le d \le \min(1-\frac{t}{2}, \frac{3t}{2}+\frac{c_1+c_2}{2})$ in the paper, which also implies $|c_1-c_2| \le t$ and $\bar{\beta} \ge \frac{2}{3}$.

The equilibrium prices shown above and resulting profits are presented by the eight cases in Proposition 1. □

Proof of Proposition 2 In order to obtain the equilibrium strategy, we firstly derive the equilibrium prices and profits of price-matching cases.

(1) Case MN

We know that $p_1^{NN} \le p_2^{NN}$ when $\beta<1$ and $\alpha \le \max(\tilde{\alpha}, \frac{\bar{\alpha}+\tilde{\alpha}}{2})$ or when $\beta=1$ (if $c_1 \le c_2$). Under such a condition, it is easy to see that the equilibrium prices in the MN case are $p_1^{MN}=p_1^{NN}$ and $p_2^{MN}=p_2^{NN}$. When $\beta<1$ and $\alpha>\max(\tilde{\alpha}, \frac{\bar{\alpha}+\tilde{\alpha}}{2})$ or when $\beta=1$ (if $c_1>c_2$), i.e., $p_1^{NN}>p_2^{NN}$, there does not exist an equilibrium price for $p_1<p_2$ in the MN case. If the equilibrium exists, p_1^{MN} would be equal to or more than p_2^{MN}. For the sake of convenience, we consider that retailer 1 would set the same price as retailer

2. Therefore, we just need to check when $\alpha > \max(\tilde{\alpha}, \frac{\bar{\alpha}+\tilde{\alpha}}{2})$ or when $\beta = 1$ (if $c_1 > c_2$), the points for which of $p_1^{MN} = p_2^{MN} \in [0,1]$ represent a stable equilibrium.

First, we can show that $p_1^{MN} = p_2^{MN} \in (d,1]$ are not stable equilibrium points, as $\pi_1^{MN} = \beta\frac{p_2^{MN}-p_1^{MN}+t}{2t}(p_1^{MN}-c_1)$ and $\frac{\partial \pi_1^{MN}}{\partial p_1^{MN}} = \beta\frac{p_2^{MN}-2p_1^{MN}+t+c_1}{2t} < 0$ always holds. We can also show that $p_1^{MN} = p_2^{MN} \in (\frac{2t}{\beta}(\alpha(1-\beta)+\frac{\beta}{2})+c_1, d]$ (if $d > \frac{2t}{\beta}(\alpha(1-\beta)+\frac{\beta}{2})+c_1$) are not stable equilibrium points. Indeed, $\pi_1^{MN} = (\alpha(1-\beta)+\beta\frac{p_2^{MN}-p_1^{MN}+t}{2t})(p_1^{MN}-c_1)$ and $\frac{\partial \pi_1^{MN}}{\partial p_1^{MN}} = \alpha(1-\beta)+\beta\frac{p_2^{MN}-2p_1^{MN}+t+c_1}{2t} < 0$, retailer 1 can always improve its profit by reducing its price. Please note that $\frac{2t}{\beta}(\alpha(1-\beta)+\frac{\beta}{2})+c_1 > p_1^{NN}$ when $\beta = 1$ (if $c_1 > c_2$) or when $\alpha(1-\beta)+\frac{\beta}{2} > \frac{1}{2}+\frac{\beta(c_2-c_1)}{4t}$, i.e., $\alpha > \frac{\bar{\alpha}+\tilde{\alpha}}{2}$.

Second, we can show that $p_1^{MN} = p_2^{MN} \in [0, \frac{2t}{\beta}(1-\alpha(1-\beta)-\frac{\beta}{2})+c_2)$ are not stable equilibrium points. As $\pi_2^{MN} = ((1-\alpha)(1-\beta)+\beta\frac{p_1^{MN}-p_2^{MN}+t}{2t})(p_2^{MN}-c_2)$ and $\frac{\partial \pi_2^{MN}}{\partial p_2^{MN}} = (1-\alpha)(1-\beta)+\beta\frac{p_1^{MN}-2p_2^{MN}+t+c_2}{2t} > 0$, retailer 2 can improve its profit by increasing the price. Please note that $\frac{2t}{\beta}(1-\alpha(1-\beta)-\frac{\beta}{2})+c_2 < p_2^{NN}$ when $\beta = 1$ (if $c_1 > c_2$) or when $\alpha(1-\beta)+\frac{\beta}{2} > \frac{1}{2}+\frac{\beta(c_2-c_1)}{4t}$,i.e, $\alpha > \frac{\bar{\alpha}+\tilde{\alpha}}{2}$.

Finally, we can show that $p_1^{MN} = p_2^{MN} \in [\frac{2t}{\beta}(1-\alpha(1-\beta)-\frac{\beta}{2})+c_2, \min(d, \frac{2t}{\beta}(\alpha(1-\beta)+\frac{\beta}{2})+c_1)]$ are stable equilibrium points, if and only if retailer 2 does not change the price to d when $d-t > p_2$,i.e., $(d-c_2)(1-\alpha)(1-\beta) < (p_2^{MN}-c_2)((1-\alpha)(1-\beta)+\frac{\beta}{2})$. Indeed, as long as $|p_1^{MN}-p_2^{MN} \le t|$, $\frac{\partial \pi_1^{MN}}{\partial p_1^{MN}} = \alpha(1-\beta)+\beta\frac{p_2^{MN}-2p_1^{MN}+t+c_1}{2t} \ge 0$ holds for any $p_1^{MN} \le p_2^{MN}$, and $\frac{\partial \pi_2^{MN}}{\partial p_2^{MN}} = (1-\alpha)(1-\beta)+\beta\frac{p_1^{MN}-2p_2^{MN}+t+c_2}{2t} \le 0$ for any $p_2^{MN} \ge p_1^{MN}$, neither retailer 1 nor retailer 2 can improve its profit by reducing or increasing the price.

Therefore, if $d \le \frac{2t}{\beta}(\alpha(1-\beta)+\frac{\beta}{2})+c_1$, i.e., $\alpha \ge \bar{\alpha}$, the optimal stable equilibrium points are $p_1^{MN} = p_2^{MN} = d$. Recall that when $\beta \le \bar{\beta}$ and $\alpha > \tilde{\alpha}$, $\alpha \ge \bar{\alpha}$ always holds. Hence, when $\beta \le \bar{\beta}$ and $\alpha > \tilde{\alpha}$ or $\beta > \bar{\beta}$ and $\alpha \ge \bar{\alpha}$, the optimal equilibrium prices are $p_1^{MN} = p_2^{MN} = d$. The resulting profits are $\pi_1^{MN} = (\alpha(1-\beta)+\frac{\beta}{2})(d-c_1)$ and $\pi_2^{MN} = ((1-\alpha)(1-\beta)+\frac{\beta}{2})(d-c_2)$. If $d > \frac{2t}{\beta}(\alpha(1-\beta)+\frac{\beta}{2})+c_1$, i.e., $\alpha < \bar{\alpha}$, we can find that $d-t < \frac{2t}{\beta}(\alpha(1-\beta)+\frac{\beta}{2})+c_1$ holds when $\beta = 1$ (if $c_1 > c_2$) or when $\alpha > \frac{\bar{\alpha}+\tilde{\alpha}}{2}$, as $d \le \frac{t}{\beta}+\frac{t}{2}+\frac{c_1+c_2}{2}$. That is, retailer 2 cannot improve its profit by changing the price to d. Hence, when $\frac{\bar{\alpha}+\tilde{\alpha}}{2} < \alpha < \bar{\alpha}$ or when $\beta = 1$ (if $c_1 > c_2$), $p_1^{MN} = p_2^{MN} = \frac{2t}{\beta}(\alpha(1-\beta)+\frac{\beta}{2})+c_1$ is the optimal stable equilibrium point, and the resulting profit is $\pi_1^{MN} = (\alpha(1-\beta)+\frac{\beta}{2})(\frac{2t}{\beta}(\alpha(1-\beta)+\frac{\beta}{2}))$ and $\pi_2^{MN} = ((1-\alpha)(1-\beta)+\frac{\beta}{2})(\frac{2t}{\beta}(\alpha(1-\beta)+\frac{\beta}{2})+c_1-c_2)$.

We summarize the optimal prices and resulting profits as follows:

(a) if $\beta = 0$, then

$$p_1^{MN} = p_2^{MN} = d,$$
$$\pi_1^{MN} = \big(\alpha(1-\beta)+\tfrac{\beta}{2}\big)(d-c_1), \pi_2^{MN} = \big((1-\alpha)(1-\beta)+\tfrac{\beta}{2}\big)(d-c_2);$$

(b) if $0 < \beta \le \bar{\beta}$, then

(i) if $\alpha < \bar{\alpha}$

$$p_1^{MN} = \frac{d}{2}+\frac{t}{\beta}\Big(\alpha(1-\beta)+\frac{\beta}{2}\Big)+\frac{c_1}{2}, p_2^{MN} = d,$$
$$\pi_1^{MN} = \frac{\beta}{2t}\Big(\frac{d-c_1}{2}+\frac{t}{\beta}\Big(\alpha(1-\beta)+\frac{\beta}{2}\Big)\Big)^2, \pi_2^{MN} = (d-c_2)\Big(1-\frac{\beta}{2t}\Big(\frac{d-c_1}{2}+\frac{t}{\beta}(\alpha(1-\beta)+\frac{\beta}{2})\Big)\Big);$$

(ii) if $\bar{\alpha} \le \alpha \le \tilde{\alpha}$

$$p_1^{MN} = p_2^{MN} = d,$$
$$\pi_1^{MN} = \big(\alpha(1-\beta)+\tfrac{\beta}{2}\big)(d-c_1), \pi_2^{MN} = \big((1-\alpha)(1-\beta)+\tfrac{\beta}{2}\big)(d-c_2);$$

(iii) if $\alpha > \tilde{\alpha}$, the results are the same as those when $\bar{\alpha} \le \alpha \le \tilde{\alpha}$.

(c) if $\bar{\beta} < \beta < 1$, then

(i) if $\alpha < 2\tilde{\alpha} - \bar{\alpha}$, the results are the same as those when $\beta \le \bar{\beta}$ and $\alpha < \bar{\alpha}$;

(ii) if $2\tilde{\alpha} - \bar{\alpha} \le \alpha \le \frac{\bar{\alpha}+\tilde{\alpha}}{2}$

$$p_1^{MN} = \frac{2t}{3\beta}\big(1+\alpha(1-\beta)+\tfrac{\beta}{2}\big)+\tfrac{2c_1+c_2}{3},\ p_2^{MN} = \frac{2t}{3\beta}\big(2-\alpha(1-\beta)-\tfrac{\beta}{2}\big)+\tfrac{c_1+2c_2}{3},$$
$$\pi_1^{MN} = \frac{\beta}{2t}\Big(\frac{2t}{3\beta}\big(1+\alpha(1-\beta)+\tfrac{\beta}{2}\big)+\tfrac{c_2-c_1}{3}\Big)^2,\ \pi_2^{MN} = \frac{\beta}{2t}\Big(\frac{2t}{3\beta}\big(2-\alpha(1-\beta)-\tfrac{\beta}{2}\big)+\tfrac{c_1-c_2}{3}\Big)^2;$$

(iii) if $\frac{\bar{\alpha}+\tilde{\alpha}}{2} < \alpha < \bar{\alpha}$

$$p_1^{MN} = p_2^{MN} = \frac{2t}{\beta}\big(\alpha(1-\beta)+\tfrac{\beta}{2}\big)+c_1,$$
$$\pi_1^{MN} = \big(\alpha(1-\beta)+\tfrac{\beta}{2}\big)\frac{2t}{\beta}\big(\alpha(1-\beta)+\tfrac{\beta}{2}\big),$$
$$\pi_2^{MN} = \big((1-\alpha)(1-\beta)+\tfrac{\beta}{2}\big)\big(\tfrac{2t}{\beta}(\alpha(1-\beta)+\tfrac{\beta}{2})+c_1-c_2\big);$$

(iv) if $\alpha \ge \bar{\alpha}$, the results are the same as those when $\beta \le \bar{\beta}$ and $\alpha \ge \tilde{\alpha}$;

(d) if $\beta = 1$, the results are either the same as those when $\bar{\beta} < \beta < 1$ and $\frac{\bar{\alpha}+\tilde{\alpha}}{2} < \alpha < \bar{\alpha}$ if $c_1 > c_2$, and can be simplified as,

$$p_1^{MN} = p_2^{MN} = t + c_1,$$
$$\pi_1^{MN} = \tfrac{t}{2},\ \pi_2^{MN} = \tfrac{t+c_1-c_2}{2}.$$

or the same as those when $\bar{\beta} < \beta < 1$ and $2\tilde{\alpha} - \bar{\alpha} \le \alpha \le \frac{\bar{\alpha}+\tilde{\alpha}}{2}$ if $c_1 \le c_2$, i.e.,

$$p_1^{MN} = t + \tfrac{2c_1+c_2}{3},\ p_2^{MN} = t + \tfrac{c_1+2c_2}{3},$$
$$\pi_1^{MN} = \tfrac{1}{2t}(t + \tfrac{c_2-c_1}{3})^2,\ \pi_2^{MN} = \tfrac{1}{2t}(t - \tfrac{c_2-c_1}{3})^2.$$

(2) Case NM

We can similarly get the equilibrium prices and profits of this case, and summarize the optimal prices and resulting profits as follows:

(a) if $\beta = 0$, then

$$p_1^{NM} = p_2^{NM} = d,$$
$$\pi_1^{NM} = \big(\alpha(1-\beta)+\tfrac{\beta}{2}\big)(d-c_1),\ \pi_2^{NM} = \big((1-\alpha)(1-\beta)+\tfrac{\beta}{2}\big)(d-c_2);$$

(b) if $0 < \beta \le \bar{\beta}$, then

(i) if $\alpha \le \bar{\alpha}$

$$p_1^{NM} = p_2^{NM} = d,$$
$$\pi_1^{NM} = \big(\alpha(1-\beta)+\tfrac{\beta}{2}\big)(d-c_1),\ \pi_2^{NM} = \big((1-\alpha)(1-\beta)+\tfrac{\beta}{2}\big)(d-c_2);$$

(ii) if $\bar{\alpha} \le \alpha \le \tilde{\alpha}$, the results are the same as those when $\alpha \le \bar{\alpha}$.

(iii) if $\alpha > \tilde{\alpha}$

$$p_1^{NM} = d, p_2^{NM} = \tfrac{d}{2} + \tfrac{t}{\beta}\big((1-\alpha)(1-\beta)+\tfrac{\beta}{2}\big)+\tfrac{c_2}{2},$$
$$\pi_1^{NM} = (d-c_1)\Big(1-\tfrac{\beta}{2t}\big(\tfrac{d-c_2}{2}+\tfrac{t}{\beta}((1-\alpha)(1-\beta)+\tfrac{\beta}{2})\big)\Big),$$
$$\pi_2^{NM} = \tfrac{\beta}{2t}\Big(\tfrac{d-c_2}{2}+\tfrac{t}{\beta}\big((1-\alpha)(1-\beta)+\tfrac{\beta}{2}\big)\Big)^2;$$

(c) if $\bar{\beta} < \beta < 1$, then

(i) if $\alpha \le \tilde{\alpha}$, the results are the same as those when $\beta \le \bar{\beta}$ and $\alpha \le \bar{\alpha}$;

(ii) if $\tilde{\alpha} < \alpha < \frac{\bar{\alpha}+\tilde{\alpha}}{2}$

$$p_1^{NM} = p_2^{NM} = \frac{2t}{\beta}\Big((1-\alpha)(1-\beta)+\frac{\beta}{2}\Big)+c_2,$$
$$\pi_1^{NM} = \Big(\alpha(1-\beta)+\frac{\beta}{2}\Big)\bigg(\frac{2t}{\beta}\Big((1-\alpha)(1-\beta)+\frac{\beta}{2}\Big)+c_2-c_1\bigg),$$
$$\pi_2^{NM} = \Big((1-\alpha)(1-\beta)+\frac{\beta}{2}\Big)\frac{2t}{\beta}\Big((1-\alpha)(1-\beta)+\frac{\beta}{2}\Big);$$

(iii) if $\frac{\bar{\alpha}+\tilde{\alpha}}{2} \le \alpha \le 2\bar{\alpha}-\tilde{\alpha}$

$$p_1^{NM} = \frac{2t}{3\beta}\Big(1+\alpha(1-\beta)+\frac{\beta}{2}\Big)+\frac{2c_1+c_2}{3},\ p_2^{NM} = \frac{2t}{3\beta}\Big(2-\alpha(1-\beta)-\frac{\beta}{2}\Big)+\frac{c_1+2c_2}{3},$$
$$\pi_1^{NM} = \frac{\beta}{2t}\bigg(\frac{2t}{3\beta}\Big(1+\alpha(1-\beta)+\frac{\beta}{2}\Big)+\frac{c_2-c_1}{3}\bigg)^2,\ \pi_2^{NM} = \frac{\beta}{2t}\bigg(\frac{2t}{3\beta}\Big(2-\alpha(1-\beta)-\frac{\beta}{2}\Big)+\frac{c_1-c_2}{3}\bigg)^2;$$

(iv) if $\alpha > 2\bar{\alpha}-\tilde{\alpha}$, the results are the same as those when $\beta \le \bar{\beta}$ and $\alpha > \tilde{\alpha}$;

(d) if $\beta = 1$, the results are either the same as those when $\bar{\beta} < \beta < 1$ and $\tilde{\alpha} < \alpha < \frac{\bar{\alpha}+\tilde{\alpha}}{2}$ if $c_2 > c_1$, and can be simplified as,

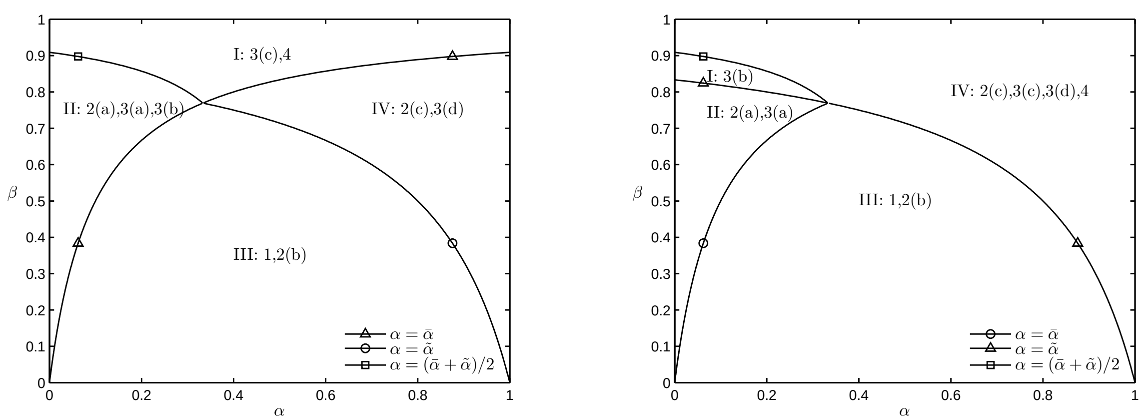
$$p_1^{NM} = p_2^{NM} = t + c_2,$$
$$\pi_1^{NM} = \frac{t+c_2-c_1}{2},\ \pi_2^{NM} = \frac{t}{2},$$

or the same as those when $\bar{\beta} < \beta < 1$ and $\frac{\bar{\alpha}+\tilde{\alpha}}{2} \le \alpha \le 2\bar{\alpha}-\tilde{\alpha}$ if $c_2 \le c_1$, i.e,

$$p_1^{NM} = t + \frac{2c_1+c_2}{3},\ p_2^{NM} = t + \frac{c_1+2c_2}{3},$$
$$\pi_1^{NM} = \frac{1}{2t}\Big(t+\frac{c_2-c_1}{3}\Big)^2,\ \pi_2^{NM} = \frac{1}{2t}\Big(t-\frac{c_2-c_1}{3}\Big)^2.$$

The different scenarios of results of cases MN and NM are visualized in Figures 7(a) and 7(b), respectively.

Figure 7 Asymmetric price-matching.



(a) Case MN (b) Case NM

Note. $c_1 = 0.1$, $c_2 = 0$, $d = 0.7$ and $t = 0.5$.

(3) Case MM

It is easy to find the optimal equilibrium prices are $p_1^{MM} = p_2^{MM} = d$ that ensures the whole market covered. Therefore, when both retailers offer price-matching, the optimal prices and profits are as follows:

$$p_1^{MM} = p_2^{MM} = d,$$
$$\pi_1^{MM} = (d-c_1)\Big(\alpha(1-\beta)+\frac{\beta}{2}\Big),\ \pi_2^{MM} = (d-c_2)\Big((1-\alpha)(1-\beta)+\frac{\beta}{2}\Big).$$

We then compare the results between the asymmetric PM cases and the base case. According to the proofs above, it is easy to see $p_1^{MN} \geq p_1^{NN}$ and $p_2^{MN} \geq p_2^{NN}$. Similarly, we have that $p_1^{NM} \geq p_1^{NN}$ and $p_2^{NM} \geq p_2^{NN}$. It is also easy to see that $\pi_1^{MN} \geq \pi_1^{NN}$ and $\pi_2^{NM} \geq \pi_2^{NN}$. In particular, $p_1^{MN} > p_1^{NN}$ and $\pi_1^{MN} > \pi_1^{NN}$ when $\alpha > \max(\frac{\tilde{\alpha}+\bar{\alpha}}{2}, \tilde{\alpha})$ or when $\beta = 1$ (if $c_1 > c_2$), and $p_2^{NM} > p_2^{NN}$ and $\pi_2^{NM} > \pi_2^{NN}$ when $\alpha < \min(\frac{\tilde{\alpha}+\bar{\alpha}}{2}, \bar{\alpha})$, or when $\beta = 1$ (if $c_1 < c_2$).

We finally compare the results between the symmetric PM case and the asymmetric PM cases. It is easy to see that $p_1^{MM} \geq p_1^{MN}$ and $p_1^{MM} \geq p_1^{NM}$, and that $p_2^{MM} \geq p_2^{MN}$ and $p_2^{MM} \geq p_2^{NM}$, because $p_1^{MM} = p_2^{MM} = d$. We next just prove that $\pi_1^{MM} \geq \pi_1^{NM}$. According to the results of asymmetric cases, we can see when $\alpha > \tilde{\alpha}$ or when $\beta = 1$, $p_2^{NM} \leq p_1^{NM} < d$, which indicates that retailer 1's market share and price in the MM case are more than those in the NM case, and thus we have $\pi_1^{MM} > \pi_1^{NM}$. When $\alpha \leq \tilde{\alpha}$, $p_1^{NM} = p_2^{NM} = d$, and apparently $\pi_1^{MM} = \pi_1^{NM}$. Therefore, we have $\pi_1^{MM} > \pi_1^{NM}$, when $\alpha > \tilde{\alpha}$ or when $\beta = 1$, and $\pi_1^{MM} = \pi_1^{NM}$, when $\alpha \leq \tilde{\alpha}$. Similarly, we can show $\pi_2^{MM} > \pi_2^{MN}$ when $\alpha < \bar{\alpha}$ or when $\beta = 1$, and $\pi_2^{MM} = \pi_2^{MN}$ when $\alpha \geq \bar{\alpha}$.

In summary, we can see when $\alpha > \tilde{\alpha}$ or when $\beta = 1$, $\pi_1^{MM} > \pi_1^{NM}$ and $\pi_1^{MN} \geq \pi_1^{NN}$, and when $\alpha \leq \tilde{\alpha}$, $\pi_1^{MM} = \pi_1^{NM}$ and $\pi_1^{MN} = \pi_1^{NN}$; when $\alpha < \bar{\alpha}$ or when $\beta = 1$, $\pi_2^{MM} > \pi_2^{MN}$ and $\pi_2^{NM} \geq \pi_2^{NN}$, and when $\alpha \geq \bar{\alpha}$, $\pi_2^{MM} = \pi_2^{MN}$ and $\pi_2^{NM} = \pi_2^{NN}$. Thus, the proposition is proved. □

Proof of Corollary 1. According to the Proof of Proposition 2, we can find $p_1^E = p_2^E = d$, and thus the parts (i) and (iii) of Corollary 1 can be proved. We next prove the part (ii) of Corollary 1. We know $\pi_1^E = (d - c_1)(\alpha(1-\beta) + \frac{\beta}{2})$ and $\pi_2^E = (d - c_2)((1-\alpha)(1-\beta) + \frac{\beta}{2})$. When $E = MM$, we can see $\pi_1^E = \pi_1^{MM} > \pi_1^{NN}$ and $\pi_2^E = \pi_2^{MM} > \pi_2^{NN}$ according to the Proof of Proposition 2. We next just show that, when $E = MN$, $\pi_2^E = \pi_2^{MN} < \pi_2^{NN}$ when $\alpha > \max(\tilde{\alpha}, \hat{\alpha})$ as follows, where $\hat{\alpha} = \frac{1}{1-\beta}(\frac{\beta(c_1-c_2-t)}{2t} - \frac{9\beta(d-c_2)}{4t} + 2 + \frac{9\beta}{4t}\sqrt{(d-c_2)^2 - \frac{8t+4\beta(c_1-c_2)}{9\beta}(d-c_2)})$.

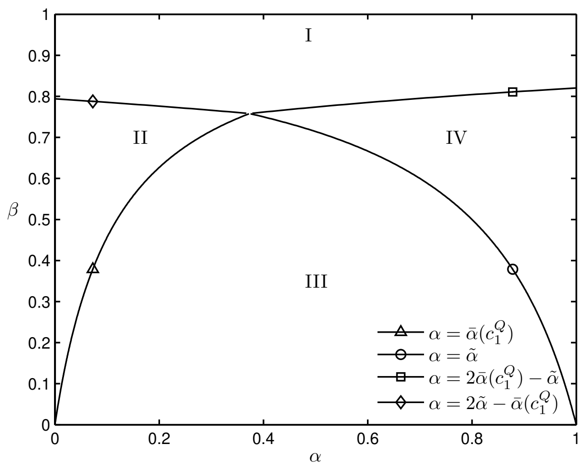
First, when $\alpha \geq \max(2\bar{\alpha} - \tilde{\alpha}, \tilde{\alpha})$, $\pi_2^{NN} - \pi_2^{MN} = \frac{t}{2\beta}(\frac{\beta(d-c_2)}{2t} + (1-\alpha)(1-\beta) + \frac{\beta}{2})^2 - ((1-\alpha)(1-\beta) + \frac{\beta}{2})(d - c_2) = \frac{t}{2\beta}(\frac{\beta(d-c_2)}{2t} - ((1-\alpha)(1-\beta) + \frac{\beta}{2}))^2 \geq 0$, where the equality holds if and only if $\alpha = \tilde{\alpha}$. Therefore, when $\beta \leq \bar{\beta}$ and $\alpha > \tilde{\alpha}$, $\pi_2^{NN} - \pi_2^{MN} > 0$; when $\beta > \bar{\beta}$ and $\alpha \geq 2\bar{\alpha} - \tilde{\alpha}$, $\pi_2^{NN} - \pi_2^{MN} > 0$.

Second, when $\bar{\alpha} \leq \alpha \leq 2\bar{\alpha} - \tilde{\alpha}$ ($\beta > \bar{\beta}$), $\pi_2^{NN} - \pi_2^{MN} = \frac{\beta}{2t}(\frac{2t}{3\beta}(1 + (1-\alpha)(1-\beta) + \frac{\beta}{2}) + \frac{c_1-c_2}{3})^2 - ((1-\alpha)(1-\beta) + \frac{\beta}{2})(d - c_2)$. Let $y = \alpha(1-\beta) + \frac{\beta}{2}$ and $f(y) = \frac{\beta}{2t}(\frac{2t}{3\beta}(2-y) + \frac{c_1-c_2}{3})^2 - (1-y)(d-c_2)$. We can see $\frac{df(y)}{d\alpha} = (1-\beta)f'(y) = (1-\beta)(-\frac{2}{3}(\frac{2t}{3\beta}(2-y) + \frac{c_1-c_2}{3}) + d - c_2) > 0$ holds for $y \geq \frac{\beta(d-c_1)}{2t}$, i.e., $\alpha \geq \bar{\alpha}$. When $\alpha = \bar{\alpha}$, i.e., $\alpha(1-\beta) + \frac{\beta}{2} = \frac{\beta(d-c_1)}{2t}$, $\pi_2^{NN} - \pi_2^{MN} = \frac{\beta}{2t}(\frac{4t}{3\beta} - \frac{d-2c_1+c_2}{3})^2 - (1 - \frac{\beta(d-c_1)}{2t})(d-c_2) = \frac{\beta}{18t}(\frac{16t^2}{\beta^2} - (16(d-c_1) + 10(d-c_2))\frac{t}{\beta} + (d-2c_1+c_2)^2 + 9(d-c_1)(d-c_2))$. Let $g(\frac{t}{\beta}) = \frac{16t^2}{\beta^2} - (16(d-c_1) + 10(d-c_2))\frac{t}{\beta} + (d-2c_1+c_2)^2 + 9(d-c_1)(d-c_2)$. We know $\frac{t}{\beta} < d - \frac{c_1+c_2}{2}$ given $\beta > \bar{\beta}$, and $\alpha(1-\beta) + \frac{\beta}{2} = \frac{\beta(d-c_1)}{2t}$ implies $\frac{t}{\beta} \geq \frac{d+t-c_1}{2}$. Because $d - c_2 \leq 2t$ given $t + \max(c_1, c_2) \leq d \leq \frac{3t}{2} + \frac{c_1+c_2}{2}$, we have $|\frac{d+t-c_1}{2} - \frac{16(d-c_1)+10(d-c_2)}{32}| < d - \frac{c_1+c_2}{2} - \frac{16(d-c_1)+10(d-c_2)}{32}$, and thus, $g(\frac{t}{\beta}) < g(d - \frac{c_1+c_2}{2}) = 0$ holds for $\frac{t}{\beta} \in [\frac{d+t-c_1}{2}, d - \frac{c_1+c_2}{2})$. Therefore, we have $\pi_2^{NN} - \pi_2^{MN} < 0$ when $\bar{\beta} < \beta < 1$ and $\alpha = \bar{\alpha}$. In addition, we have shown that $\pi_2^{NN} - \pi_2^{MN} > 0$ when $\beta > \bar{\beta}$ and $\alpha = 2\bar{\alpha} - \tilde{\alpha}$. Solving $f(y) = 0$ yields the bigger root $y = 2 + \frac{\beta(c_1-c_2)}{2t} - \frac{9\beta(d-c_2)}{4t} + \frac{9\beta}{4t}\sqrt{(d-c_2)^2 - (\frac{8t+4\beta(c_1-c_2)}{9\beta})(d-c_2)}$, i.e., $\alpha = \hat{\alpha}$, and we know $\bar{\alpha} \leq \hat{\alpha} \leq 2\bar{\alpha} - \tilde{\alpha}$. That is, when $\alpha > \hat{\alpha}$, $\pi_2^{NN} - \pi_2^{MN} > 0$.

Therefore, $\pi_2^{MN} < \pi_2^{NN}$ when $\alpha > \hat{\alpha}$ and $\beta > \bar{\beta}$, and when $\alpha > \tilde{\alpha}$ and $\beta \leq \bar{\beta}$. We can similarly show that $\pi_1^{NM} < \pi_1^{NN}$ when $\alpha < \check{\alpha}$ ($2\tilde{\alpha} - \bar{\alpha} \leq \check{\alpha} \leq \tilde{\alpha}$) and $\beta > \bar{\beta}$, and when $\alpha < \bar{\alpha}$ and $\beta \leq \bar{\beta}$, where $\check{\alpha} = \frac{1}{1-\beta}(\frac{\beta(c_1-c_2-t)}{2t} + \frac{9\beta(d-c_1)}{4t} - 1 - \frac{9\beta}{4t}\sqrt{(d-c_1)^2 - \frac{8t+4\beta(c_2-c_1)}{9\beta}(d-c_1)})$. □

Proof of Proposition 3. Taking the first order derivative of π_1^{QI} with respect to q and p_1^{QI} yields $\frac{\partial \pi_1^{QI}}{\partial q} = \beta \frac{p_1^{QI}-c_1-c(q)}{2t}\frac{du(q)}{dq} - (\alpha(1-\beta) + \beta\frac{p_2^{QI}-p_1^{QI}+t+u(q)}{2t})\frac{dc(q)}{dq}$, and $\frac{\partial \pi_1^{QI}}{\partial p_1^{QI}} = \alpha(1-\beta) + \beta\frac{p_2^{QI}-p_1^{QI}+t+u(q)}{2t} - \beta\frac{p_1^{QI}-c_1-c(q)}{2t}$. We can easily find $\frac{du(q)}{dq} = \frac{dc(q)}{dq}$ is the necessary and sufficient condition for $\frac{\partial \pi_1^{QI}}{\partial q} = 0$ and $\frac{\partial \pi_1^{QI}}{\partial p_1^{QI}} = 0$, and that derives the optimal quality improvement level q^*. Then, according to Lemma 1, the equilibrium prices and resulting profits can be derived by letting $p_1^{NN} = p_1^{QI} - u(q^*)$ and $p_2^{NN} = p_2^{QI}$, and substituting c_1 with $c_1^{QI} = c_1 + c(q^*) - u(q^*)$.

Figure 8 Solutions to QI Case.



Note. $c_1 = 0.1$, $c_2 = 0$, $d = 0.7$, $t = 0.5$, $c(q) = 1.5q^2$, and $u(q) = 0.1q^{1/2}$.

Similar to Figure 2, Figure 8 visualizes four regions for the results of the case. □

Proof of Corollary 2. Observing the results shown in Proposition 1, we can see p_1^{NN} and π_2^{NN} increase in c_1, while p_2^{NN} and π_1^{NN} decrease in c_1. Therefore, we have $p_2^{QI} = p_2^{NN}(c_1^{QI}) > p_2^{NN}$, $\pi_1^{QI} = \pi_1^{NN}(c_1^{QI}) > \pi_1^{NN}$, and $\pi_2^{QI} = \pi_2^{NN}(c_1^{QI}) < \pi_2^{NN}$ according to Proposition 3, and we can also see $p_1^{NN}(c_1^{QI}) < p_1^{NN}$. Through further comparison, we can always find $p_1^{QI} = p_1^{NN}(c_1^{QI}) + u(q^*) > p_1^{NN}$. In addition, it is easy to see the consumer surplus is higher because the effective prices are lower. □

Proof of Proposition 4. The proof can be shown by comparing π_1^E with π_1^{QI} when $\alpha > \tilde{\alpha}$ or when $\alpha > 2\tilde{\alpha} - \bar{\alpha}(c_1^{QI})$, because we can see $\pi_1^{QI} \geq (d - c_1^{QI})(\alpha(1-\beta) + \frac{\beta}{2}) > \pi_1^E$ otherwise. The threshold value $\beta^{QI}(\alpha)$ can be solved by letting $\pi_1^E - \pi_1^{QI} = 0$, i.e.,

$$(d-c_1)(\alpha(1-\beta)+\frac{\beta}{2}) - (d-c_1^{QI})\Big(1 - \frac{\beta}{2t}(\frac{d-c_2}{2} + \frac{t}{\beta}((1-\alpha)(1-\beta)+\frac{\beta}{2}))\Big) = 0, \tag{11}$$

when $\alpha > 2\bar{\alpha}(c_1^{QI}) - \tilde{\alpha}$, and

$$(d-c_1)(\alpha(1-\beta)+\frac{\beta}{2}) - \frac{\beta}{2t}\Big(\frac{2t}{3\beta}(1+\alpha(1-\beta)+\frac{\beta}{2}) + \frac{c_2 - c_1^{QI}}{3}\Big)^2 = 0 \tag{12}$$

when $\alpha \leq 2\bar{\alpha}(c_1^{QI}) - \tilde{\alpha}$. Apparently, Eq. (11) has only one solution at most in the feasible region. We can show the left side of Eq. (12) always increases in β so that Eq. (12) also has no more than one solution for $\beta \leq 1$.

When $\alpha \le 1/2$, $\pi_1^E = (d - c_1)(\alpha(1-\beta) + \frac{\beta}{2})$ increases in β and $\pi_1^{QI} = \frac{\beta}{2t}\left(\frac{2t}{3\beta}\left(1 + \alpha(1-\beta) + \frac{\beta}{2}\right) + \frac{c_2 - c_1^{QI}}{3}\right)^2$ decreases in β. When $\alpha > 1/2$, we can find $\frac{d^2(\pi_1^E - \pi_1^{QI})}{d\beta^2} > 0$, and we can further see $\frac{d(\pi_1^E - \pi_1^{QI})}{d\beta} < 0$ when $\beta = 1$ given $t + \max(c_1, c_2) \le d \le \min(1 - \frac{t}{2}, \frac{3t}{2} + \frac{c_1^{QI} + c_2}{2})$. Therefore, the threshold value $\beta^{QI}(\alpha)$ can be uniquely solved. □

Proof of Corollary 3 The proof can be directly obtained by examining the equilibrium results under the optimal strategy. □