

Smart Procurement of Naturally Generated Energy (SPONGE) for Plug-in Hybrid Electric Buses

Joe Naoum-Sawaya, Emanuele Crisostomi, Mingming Liu, Yingqi Gu, and Robert Shorten

Abstract—We discuss a recently introduced ECO-driving concept known as SPONGE in the context of Plug-in Hybrid Electric Buses (PHEB)’s. Examples are given to illustrate the benefits of this approach to ECO-driving. This includes, financial, environmental, as well as demand side management benefits. Finally, distributed algorithms to realise SPONGE are discussed, paying attention to the privacy implications of solving the underlying optimisation problems.

Index Terms—Distributed algorithms, Optimisation, Plug-in hybrid electric buses, Demand side management

I. INTRODUCTION

We discuss a recently introduced holistic ECO-driving concept known as SPONGE in the context of Plug-in Hybrid Electric Buses (PHEB)’s. PHEB’s are increasingly seen as an effective tool in combating air pollution in our cities, and as a tool for reducing our cities reliance on fossil fuels (thereby reducing greenhouse gas emissions) [1], [2]¹. Consequently, the design and operation of such buses has been the subject of much research interest over recent years. Hitherto, significant research effort has focussed on improving the fuel economy while guaranteeing that both the engine and the electric machine work in the high-efficiency area; typically, by taking into account knowledge of both bus routes and passenger loadings in a predictive manner. Examples of work in this direction can be found in [3]–[5].

Our objective in this paper is to extend this line of enquiry further. Our basic setting is to consider a bus operator that has access to a fixed amount of renewable energy on a daily basis. For example, some operators may own solar farms or have access to wind generation. It makes sense to use this energy before consuming electrical energy that is bought from the grid, and in situations where there is an oversubscription for this *free* energy, the operator then has a choice as to how this energy is distributed to each bus. For example, some drivers may be more efficient than others. Thus, it makes eminent economic sense, to distribute this free energy to reduce the impact of less efficient drivers, while at the same time ensuring

that sufficient energy is consumed to make room for every that arrives the next time the buses recharge. Specifically, SPONGE for buses operates as follows.

- A. We introduce the forecast of generation of energy from renewable resources on a day ahead basis as a further variable to influence the energy management system for the bus operator.
- B. We use this forecast to prioritise the manner in which individual buses dissipate electrical energy.
- C. We do this by prioritising the utilisation of energy from renewable sources over other resources, and by taking account of the fact that some drivers/routes are more energy efficient than others.

Prioritising energy from renewable sources in this manner introduces a number of benefits for the bus operator and society.

- The use of energy from renewable sources (e.g., wind turbines, dynamic water power, or solar power) achieves environmental health benefits with respect to the use of the “power grid average” electricity [6].
- Financial benefits for the bus operator.
- Depleting PHEBs’ batteries of a pre-specified quantity of energy allows better grid-demand balancing. That is, the energy provider knows in advance how much energy will be required by PHEBs, when connected for charging. This makes the electrical load of PHEBs to be fully predictable and dispatchable, thus mitigating the burden of the power grid to accommodate a not-known-in-advance electrical load.

Note that the proposed perspective of the PHEBs charging problem closely resembles the widely discussed practises of demand side management, where electricity customers shift their electrical loads taking into account the expected availability of energy from renewable sources (e.g., solar panels on the roofs of their houses). In fact, in this paper we are considering the possibility that buses accommodate the consumption of their batteries considering the amount of energy that will be available from renewable sources when recharging.

This paper extends previous work of some of the authors in [7] for the case of electric cars. While the basic idea of matching energy from renewable sources with space in the battery of the EVs remains the same, the case of PHEBs substantially differs from the case of Plug-in Hybrid Electric Vehicles (PHEVs) in several ways: (i) the optimisation procedure where the driving mode is associated with a bus is a computed in a batch fashion; (ii) a-priori knowledge of the bus routes is used in the optimisation formulation; (iii) privacy

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¹See <https://chargedevs.com/newswire/ultramodern-plug-in-buses-go-into-service-in-göthenburg/> for further examples.

preservation of two competitive optimisation algorithms is discussed; and (iv) a brief discussion of the accuracy of the forecasts of power generation from renewable sources is given.

II. SPONGE PROBLEM FORMULATION

Let $\mathcal{N} = \{1, 2, \dots, N\}$ denote the set of N PHEBs participating to the SPONGE programme. We shall make the following assumptions:

- We assume that after a number of trips along their (different) routes, the N PHEBs stop for charging at the bus station. For instance, we can assume that the PHEBs will not drive from 11pm to 6am, and they will be charged in this time frame;
- We also assume that a 24-hour ahead forecast of energy from the renewable energy sources available to the operator will be available as well (e.g., a forecast of how much energy will be generated by the wind plants connected with the charging station at night time). We denote this amount of energy available by E_{av} ;
- Early in the morning, before being dispatched along their routes, the buses will compute how the energy E_{av} should be optimally shared among themselves during the day (i.e., in terms of energy consumption of their own batteries);
- In order to compute the optimal allocations of energy, we shall assume that each PHEB is equipped with a device to transmit messages to the central infrastructure via Vehicle-to-Infrastructure (V2I) technology;
- The central infrastructure has the ability to broadcast messages to the whole network of PHEBs using some Infrastructure-to-Vehicles (I2V) technology.

Note that in our set-up we shall not require vehicles to exchange information among themselves, and thus, we shall not require PHEBs to be equipped with Vehicle-to-Vehicle (V2V) communication devices. A schematic diagram of the above SPONGE paradigm is illustrated in Fig. 1.

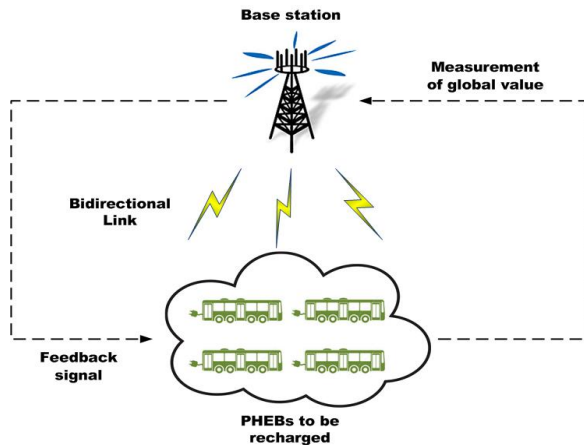


Fig. 1: A schematic diagram of the SPONGE programme

When travelling along their routes, the buses will be able to choose when it is more convenient to switch from electric mode to ICE mode (i.e., using the Internal Combustion

Engine) and back. In this context, we denote by d_i the energy consumption by the i 'th bus along its trip. Then we are interested in computing the solution of the optimisation problem as follows:

$$\begin{cases} \max_{d_1, d_2, \dots, d_N} \sum_{i \in \mathcal{N}} f_i(d_i) \\ \text{s.t.} \sum_{i \in \mathcal{N}} d_i = E_{av} \end{cases} \quad (1)$$

In the optimisation problem (1), the terms d_i can be interpreted as a ‘‘budget’’ of energy that is allocated to the i 'th bus in order to maximise a utility function of interest, such that the sum of the energy budgets allocated to all the buses matches E_{av} as in the SPONGE spirit. Although in principle the utility function $\sum_{i \in \mathcal{N}} f_i(d_i)$ may be chosen in an arbitrary fashion, to represent any *utility*, in this work we shall explore the particular case where one is interested in the utility of CO_2 emissions $f_i(d_i)$ as achieved by each bus. Clearly, each $f_i(d_i)$ is an increasing function of d_i as no CO_2 emissions are saved when the bus travels all the time in ICE mode, while the emissions-per-trip quantity are saved when the vehicles travel in electric mode all the time. The utility functions of 15 PHEBs that we shall study are shown in Fig. 2 as a function of the percentage of the use of the electrical engine for each bus. These functions are constructed from real data and the next section will explain how the utility functions are designed in detail. In addition, Section III will describe an attractive distributed solution of the optimisation problem (1).

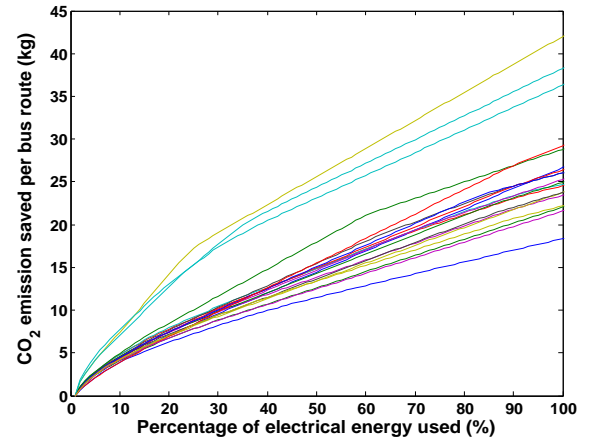


Fig. 2: Utility functions of 15 PHEBs in Dublin city.

A. Construction of the utility function

Electrical energy consumption: Under the assumption that a vehicle is travelling at a constant speed s , the amount of electrical energy consumption of vehicles can be modelled as a convex function of s , see [8], [9] for instance. The convex function depends in turn on, among other things, the physical characteristics of the bus. In our work, we used the real energy consumption data of a BYD electric bus [9] and noticed that it can be accurately approximated with a quadratic function of the vehicle speed s as

$$e(s) = \alpha_0 s^2 + \alpha_1 s + \alpha_2. \quad (2)$$

where $\alpha_0, \alpha_1, \alpha_2$ are all constant parameters. Using a conventional least square method to fit the real energy consumption we obtained α_0, α_1 and α_2 equal to $\frac{22}{28777}, \frac{-213}{2599}$ and $\frac{2384}{783}$, respectively, and the corresponding utility function is depicted in Fig. 3. In particular, note that the energy is large when the speed is large, which is caused by the fact that power consumption increases with the cube of the speed for aerodynamic reasons, and that the energy is large again when the speed is very low, due to the fact that travel times increase.

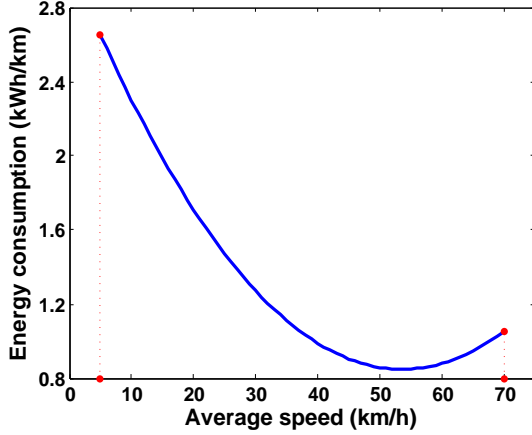


Fig. 3: A typical energy cost function for PHEBs.

Saving of CO_2 : Analogously to power consumption, also CO_2 emissions can be computed as a function of the speed of the vehicles, by adopting for instance the following well-known average-speed model (from [10]) as

$$h(s) = k \left(\frac{a + bs_i + cs^2 + ds^3 + es^4 + fs^5 + gs^6}{s} \right), \quad (3)$$

where $a, b, c, d, e, f, g, k \in \mathbb{R}$ are used to specify different levels of emissions by different classes of vehicles. In particular, in our work we used the cost function depicted in Fig. 4, that corresponds to the vehicle code R203 in [10] (i.e., diesel buses with up to 15 tonnes of gross vehicle mass).

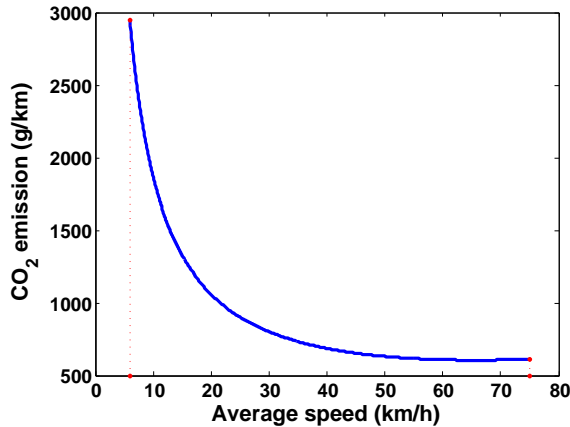


Fig. 4: CO_2 emission cost function for PHEBs.

Utility functions f_i : As anticipated in Section II, the overall utility function $f_i(d_i)$ quantifies how much CO_2 has been saved by the i 'th bus, provided that the bus is allowed to spend a budget of d_i units of energy along its route. In the following, we shall assume that each route can be split into a number of very small sections, corresponding to the distance traveled by a bus in one second. For simplicity, we shall further assume that speed limits do not change within sections, and that a bus will travel at a speed equal to the speed limit within the section. Finally, we shall denote by \mathcal{R}_i the set of all sections along the route of the i 'th PHEB, and by γ_l the fraction of the time that a PHEB will travel in EV mode along the l 'th section of the route. Thanks to the fact that all bus routes are fixed and known a priori, then the utility functions can be computed off-line by optimally deploying the energy budget d_i along the routes as follows:

$$\begin{cases} f_i(d_i) = \max_{\gamma_l} \sum_{l \in \mathcal{R}_i} h(s_{\max}(l)) \cdot \gamma_l \\ \text{s.t.} \sum_{l \in \mathcal{R}_i} e(s_{\max}(l)) \cdot L(l) \cdot \gamma_l = d_i \\ 0 \leq \gamma_l \leq 1, l \in \mathcal{R}_i \end{cases} \quad (4)$$

where $L(l)$ denotes the length of the l 'th section of a trip.

Lemma 1 *Functions $f_i(d_i)$ are concave.*

A proof of this lemma is given in the Appendix.

Roughly speaking, the utility functions describe the maximum CO_2 savings that can be achieved by a bus, given that the bus is allocated a budget of d_i units of energy that can be spent in travelling in electric mode. The knowledge of the route is used to optimally decide when it is best to drive in ICE mode and when in electric mode in order to maximise CO_2 savings without exceeding the energy budget. Built on such utility functions, the next section will show how to optimally allocate the expected energy E_{av} into the single budgets.

III. ALGORITHMS AND OPTIMAL SOLUTION

The SPONGE idea consists in optimally allocating budgets of energy to single buses, in order to overall match the expected energy generated from renewable sources during the charging process. In principle, many different methods may be used to solve the optimisation problem (1) that arises in this context, for instance ADMM-like algorithms [11]. ADMM (Alternating Direction Method of Multipliers) is a popular optimisation algorithm, that has been recently proposed as an evolution of other well-known optimisation algorithms, like the dual ascent and the method of multipliers. As an alternative to ADMM-like algorithms, our choice here is to adopt an AIMD-like algorithm [12] to solve the problem in a distributed fashion. Such a choice is motivated by many reasons:

- **Low-communication requirements:** Although for simplicity we presented here a simple case study with (possibly) few buses, the same programme can be easily generalised to include tens of buses, possibly including

suburban and extraurban buses of different companies as well. Also, the batch optimisation formulation might be solved in real-time to account for non fully-predictable aspects, as for instance traffic peaks or the receipt of weather forecast updates. In this perspective, it is convenient to maintain the quantity of exchanged information as low as possible, to improve the scalability of the proposed solution.

- **Privacy-preservation requirements:** Again, the simple problem presented here, where we assumed that all buses would pollute or consume energy in the same fashion (i.e., same functions $h(\cdot)$ and $e(\cdot)$ for all buses in Section II-A), can be easily generalised to consider more realistic situations. In practice, energy consumption or pollution depend on other factors as well, as for instance the weight of the buses (and thus, the number of passengers on-board), different driving styles of the drivers, and a possibly different use of electrical ancillary services (e.g., air conditioning). In this context, drivers might prefer not to share their own utility functions (to avoid being evaluated on their, possibly not environmentally friendly, driving attitudes), or companies might prefer not to reveal private information (e.g., number of passenger on-board).
- **Agent actuation:** AIMD requires very little actuation ability on the agent-side. This is in contrast to ADMM where at each time step, agents must solve a local optimisation problem.
- **Algorithm parameterisation:** In AIMD, it is easily shown that the gain parameters of the network are independent of network dimension; rather, they only depend on the largest derivative over all utility functions. Thus, selecting a gain for the algorithm is extremely simple in the case of AIMD.

As we shall further discuss in the following section, AIMD is a convenient alternative to ADMM, when the previous aspects are relevant.

A. AIMD Algorithm

Additive Increase Multiplicative Decrease (AIMD) algorithms were originally applied for solving issues arising in network congestion in the Internet [13]. To date, this idea has been widely explored for the design of practical algorithms for other applications as well, as for instance, network applications see [14]–[16], and smart grid applications see [17]–[19]. More recently, an unsynchronised AIMD algorithm based on the nonhomogeneous place-dependent Markov chains model was proposed in [12] to solve utility optimisation problems. The pseudo-code of the proposed algorithm is given in Algorithm 1. Note that the algorithm does not compute the optimal budgets d_i in a single step, but in an iterative fashion, as $d_i(k)$ represents the value of the unknown energy to be allocated to the i 'th PHEB, computed at time step k . For large values of k , $d_i(k)$ will eventually converge to the optimal solution that maximises the environmental benefits (while still satisfying

Algorithm 1 Unsynchronised AIMD Algorithm

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1: Initialisation:  $k = 1$ ,  $d_i(k) = 0$ ;
2: Broadcast the parameter  $\Gamma$  to the entire networks;
3: while  $k < k_{\max}$  do
4:   if  $\sum_{i=1}^N d_i(k) < E_{av}$  then
5:      $d_i(k+1) = d_i(k) + \alpha$ 
6:   else if with probability  $p_i(k) = \Gamma \frac{1}{\bar{d}_i(k) f'_i(\bar{d}_i(k))}$  then
7:      $d_i(k+1) = \beta d_i(k)$ 
8:   else
9:      $d_i(k+1) = d_i(k) + \alpha$ 
10:  end if
11:   $k = k + 1$ 
12: end while

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the energy constraint). In Algorithm 1, k_{\max} represents the maximum number of iterations before the algorithm stops (e.g., after five minutes of iterations).

The basic idea of Algorithm 1 is that if the sum of the $d_i(k)$ of all PHEBs is smaller than E_{av} , then each PHEB increases its target energy consumption $d_i(k)$ at the next iteration $k+1$ by a quantity α . However, if the sum of the energy budgets of all PHEVs exceeds E_{av} (this situation is usually called as a congestion event), then each PHEB decreases its energy consumption by a multiplicative factor $0 < \beta < 1$ with probability $p_i(k) = \Gamma \frac{1}{\bar{d}_i(k) f'_i(\bar{d}_i(k))}$, where Γ is a constant common parameter broadcast by the CA, and $\bar{d}_i(k)$ is the time average of the sequence of $d_i(k)$ at congestion events, up to the last iteration. It is proved in [12] that $\bar{d}_i(k)$ approaches to the optimal solution of the problem when Algorithm 1 converges and where the optimisation is carried out over the $f_i(\bar{d}_i)$.

Remark (AIMD): The philosophy underlying the AIMD algorithm is to adjust $p_i(k)$ and $d_i(k)$ at every time step k such that for large values of k , $f'_i(\bar{d}_i(k)) = f'_j(\bar{d}_j(k))$, $\forall i \neq j \in \mathcal{N}$, or in other words the PHEBs achieve consensus on the derivatives of their utility functions. This, with strict convexity of the utility functions, is both necessary and sufficient for optimality. This property is known from elementary optimisation theory. Algorithm 1 was originally designed in [12] to *minimise* a cost function of interest, here we slightly adapt it to *maximise* CO_2 savings. Accordingly, given that each utility function f_i in our case is strictly concave, and that the p_i are strictly non-increasing in our problem, then we can adapt the algorithm in [12] so that consensus is achieved on $1/f'_i$, and the convergence and optimality properties of the algorithm are preserved.

Remark (Privacy): In our application, the utility functions $f_i(d)$ potentially reveal sensitive private information. For example, as formulated, these functions reveal how good a particular driver is on a given route. This information is potentially very useful for an employer and could potentially be used in a nefarious manner. In addition, in unionised environments, revealing these functions to an employer could also be of concern and consequently impede the adaptation

is ideas like SPONGE. Given this context, a natural question is whether the distributed optimisation can be solved without revealing private information. We now make some brief comments in this direction. Recall that we assume that the central agent may receive the value d_i from agent i , and performs the aggregation $A = \sum_{i=1}^N d_i$. We also assume that there are no incentives for an agent to cooperate with the central agent to help deduce the f'_i ; that is, all agents, other than the central agent, are honest. Given this basic setting, roughly speaking, one may discern the following four levels of privacy when speaking about our utility minimization problem.

- (i) Absolute utility privacy (AUP) : Here, the central agent cannot deduce $f_i(d)$ based on knowledge available to it. This is a basic level of privacy.
- (ii) Relative utility privacy (RUP) : Here the central agent cannot deduce whether $f_i(d) > f_j(d)$. This again, is a basic level of privacy.
- (iii) Absolute derivative privacy (ADP) : Here, the central agent cannot deduce $f'_i(d)$ based on knowledge available to it. This information is important since it allows the central agents to estimate the *price* elasticity of individual agents.
- (iv) Relative derivative privacy (RDP) : Here the central agent cannot deduce whether $f'_i(d) > f'_j(d)$.

Privacy preservation is beyond the scope of this paper. However, we note briefly that the stochastic AIMD algorithm allows us to give some guarantees regarding all of these privacy categories. First since the optimisation is based on $f'_i(d_i)$, the AIMD algorithm can be considered *AUP*- and *ADP*-private. Deducing any $f'_i(d_i)$ would require estimation of the $p_i(k)$. Clearly, this is difficult (but not impossible) except at optimal points. However, since our algorithm only requires an implicit consensus among all derivatives, one may replace in the formula for $p_i(k)f'_i(d_i)$ with $g_i(f'_i(d_i))$, where g_i is chosen so that the convergence conditions in [12] are satisfied. Clearly, without knowledge of each function g_i , the central agent cannot deduce $f'_i(d_i)$ even if the probabilities p_i 's are correctly estimated when the algorithm converges.

IV. SUMO SIMULATIONS

A. Simulation Set-up

In this section, we evaluate the performance of the proposed AIMD algorithm in a realistic traffic scenario, where vehicular flows are simulated using the popular mobility simulator SUMO [20]. In doing so, we shall also compare the results obtained using AIMD with those obtained with the ADMM algorithm. Finally, we shall also shortly discuss the impact of inaccurate predictions made by weather forecasting tools (e.g., from solar PV panels). All the simulations are performed over the road network of Dublin, Ireland, depicted in Fig. 5, imported from OpenStreetMap [21].



Fig. 5: Road network of Dublin City, Ireland, imported from OpenStreetMap, used in our simulations.

B. Simulation Results

We assume that 15 PHEBs participate to a SPONGE programme in Dublin city, Ireland. We further assume that weather forecasting tools predict an availability 250 kWh in the next charging period. Ten minutes before starting their routes, the buses and the CA solve the optimisation problem using the described AIMD algorithm, and optimally allocate the 250 kWh among the different buses in terms of energy budgets. Then, we assume that the PHEBs have an hour of time to spend their budgets of energy. Fig. 6 compares the overall energy that would be required to travel every single route fully in EV mode (blue bars) with the allocated budgets (red bars). Fig. 7 shows that the AIMD algorithms eventually converges to the optimal solution (that can be easily computed exactly, assuming a full knowledge of all the utility functions), and Fig. 8 reveals that the necessary condition for optimality (KKT) has been achieved when algorithm converges (i.e., the derivatives of the utility functions converge to the same value).

Fig. 9-11 further show the optimal switches between ICE and EV mode for buses 1 and 2 along their routes for different values of expected available energy. Obviously, when the expected energy is lower, the green sections corresponding to EV motion are shorter than when the expected energy is larger.

[Will add some discussion here for results from Figs. 9 - 11](#)

C. Solar Energy

Apart from using different algorithms for optimisation, one of the most important factors for the SPONGE programme is to have a precise predication of the forthcoming available energy E_{av} . Conventionally, getting E_{av} amount of energy is not a great challenging for the grid, e.g., these energy can usually be directly bought from the day-ahead electricity market. However, in the future smart grid, where the amount of energy provided might be highly uncertain, e.g., energies are mostly provided by renewable sources close to the parking station of PHEBs, the energy predication task will be more significant for implementation of the SPONGE programme. At present,

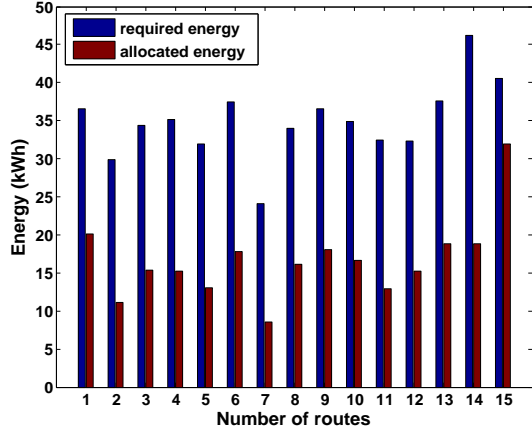


Fig. 6: Comparison of the total energy and the optimal required energy for 15 routes.

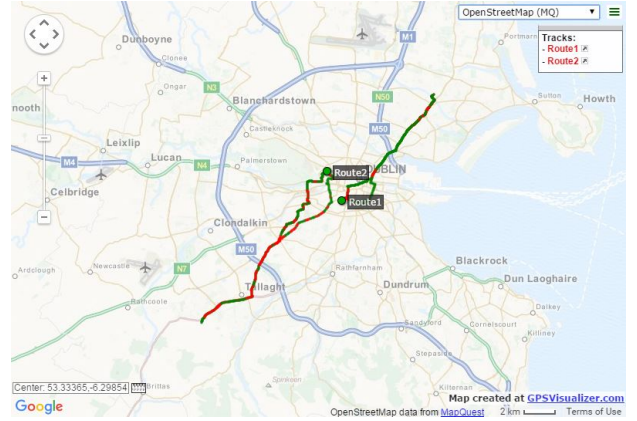


Fig. 9: Electrical energy distribution of two routes when $E_{av} = 250$ kWh. Green lines indicate PHEBs driving in EV mode and red lines indicate PHEBs in ICE mode.

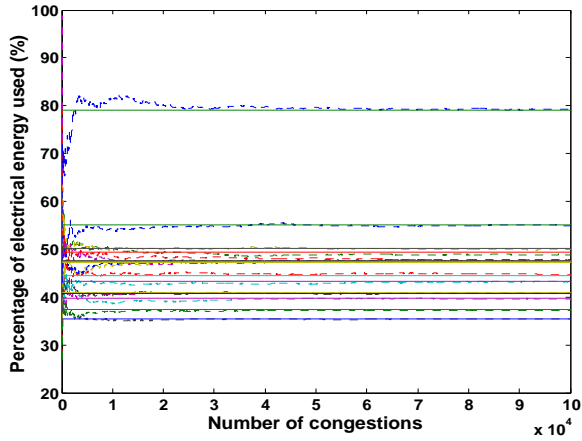


Fig. 7: Convergence to the optimal solution using AIMD

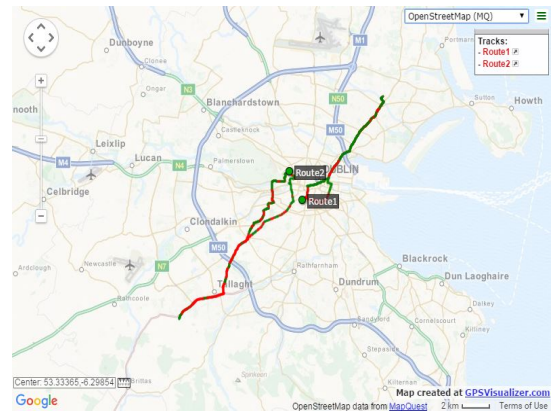


Fig. 10: Electrical energy distribution of two routes when $E_{av} = 100$ kWh.

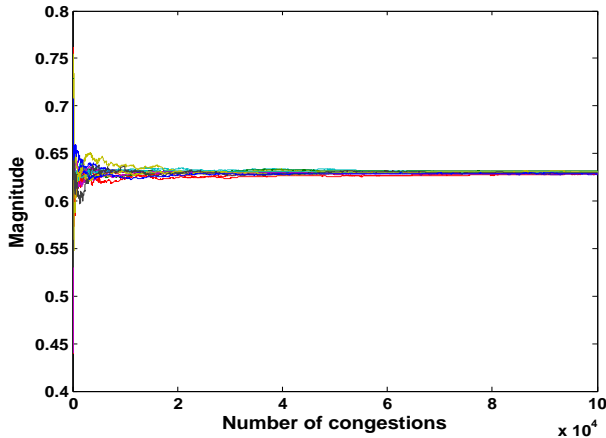


Fig. 8: Evolution of $f'_i(\bar{d}_i(k))$ s converges to consensus

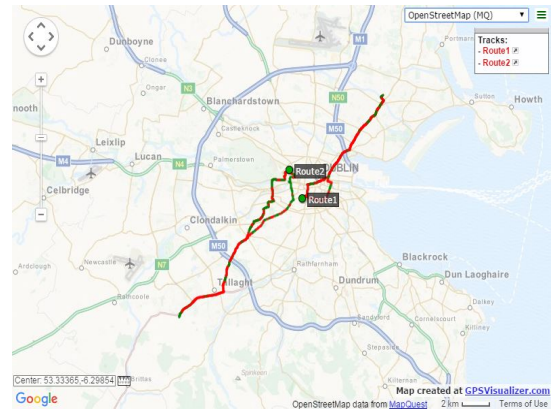


Fig. 11: Electrical energy distribution of two routes when $E_{av} = 50$ kWh.

the energy predication task is usually implemented by using accurate weather forecast engine and proper energy forecast model. However, most of these tools available nowadays are mostly commercialised and not easily accessible for public. In contrast, thanks to a few open-access online predication tools, e.g., the API provided by [22], and now we are able to predict the solar energy generation worldwide by simply providing the technical parameters required. Our objective in this section is to demonstrate the accurate level of the tool [22] by comparing the results obtained from both predication data and the real data we collected on-site. In particular, we implemented two real case studies in two different scenarios: one in a workplace scenario, where the PV panels are mounted on the flat roof of the building in University College Dublin, Ireland, and the other in a home scenario, where the PV panels are fitted on sloped roofs of the house in Pisa, Italy. Some preliminary results are demonstrated in Fig. 12 and Fig. 13. As can be seen from Fig. 12, twelve out of fourteen samples (85.71%) are lied within the area of 30% of predication accuracy in the UCD case. Similarly, in Fig. 13 it shows that 80% of samples are lied within the area of 20% predication accuracy in the Pisa case. Both results suggest that the introduced solar predication tool [22] can indeed provide a promising predication accuracy for better getting knowledge of the available energy for the SPONGE programme.

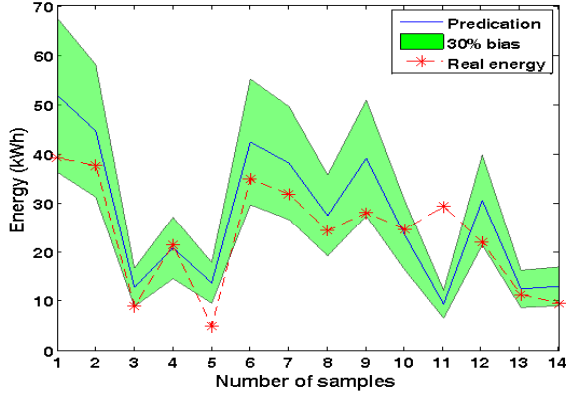


Fig. 12: Real and predicated solar energy generation in UCD.

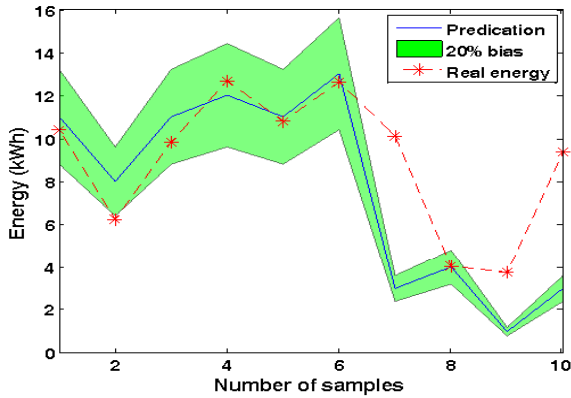


Fig. 13: Real and predicated solar energy generation in Pisa.

V. CONCLUSION

Concluding the paper...

APPENDIX

OUTLINE OF PROOF OF LEMMA 1

In this section, we give a proof for concavity of the utility function f_i in Subsection II-A. By definition (4)

$$\begin{cases} f_i(d_i) = \max_{\gamma_l} \sum_{l \in \mathcal{R}_i} h(s_{\max}(l)) \cdot \gamma_l \\ \text{s.t.} \sum_{l \in \mathcal{R}_i} e(s_{\max}(l)) \cdot L(l) \cdot \gamma_l = d_i \\ 0 \leq \gamma_l \leq 1, l \in \mathcal{R}_i \end{cases} \quad (5)$$

Consider two variables $d_i^1 \neq d_i^2$, we can show that

$$\begin{cases} f_i(d_i^1) = \max_{\gamma_l} \sum_{l \in \mathcal{R}_i} h(s_{\max}(l)) \cdot \gamma_l \\ \text{s.t.} \sum_{l \in \mathcal{R}_i} e(s_{\max}(l)) \cdot L(l) \cdot \gamma_l = d_i^1 \\ 0 \leq \gamma_l \leq 1, l \in \mathcal{R}_i \end{cases} \quad (6)$$

it then follows that

$$\begin{cases} \lambda f_i(d_i^1) = \max_{\gamma_l} \sum_{l \in \mathcal{R}_i} h(s_{\max}(l)) \cdot \lambda \gamma_l \\ \text{s.t.} \sum_{l \in \mathcal{R}_i} e(s_{\max}(l)) \cdot L(l) \cdot \gamma_l = d_i^1 \\ 0 \leq \gamma_l \leq 1, l \in \mathcal{R}_i \end{cases} \quad (7)$$

and

$$\begin{cases} \lambda f_i(d_i^1) = \max_{\bar{\gamma}_l} \sum_{l \in \mathcal{R}_i} h(s_{\max}(l)) \cdot \bar{\gamma}_l \\ \text{s.t.} \sum_{l \in \mathcal{R}_i} e(s_{\max}(l)) \cdot L(l) \cdot \frac{\bar{\gamma}_l}{\lambda} = d_i^1 \\ 0 \leq \bar{\gamma}_l \leq \lambda, l \in \mathcal{R}_i \end{cases} \quad (8)$$

Thus,

$$\begin{cases} \lambda f_i(d_i^1) = \max_{\bar{\gamma}_l} \sum_{l \in \mathcal{R}_i} h(s_{\max}(l)) \cdot \bar{\gamma}_l \\ \text{s.t.} \sum_{l \in \mathcal{R}_i} e(s_{\max}(l)) \cdot L(l) \cdot \bar{\gamma}_l = \lambda d_i^1 \\ 0 \leq \bar{\gamma}_l \leq \lambda, l \in \mathcal{R}_i \end{cases} \quad (9)$$

Similarly, we can show that

$$\begin{cases} (1 - \lambda) f_i(d_i^2) = \max_{\hat{\gamma}_l} \sum_{l \in \mathcal{R}_i} h(s_{\max}(l)) \cdot \hat{\gamma}_l \\ \text{s.t.} \sum_{l \in \mathcal{R}_i} e(s_{\max}(l)) \cdot L(l) \cdot \hat{\gamma}_l = (1 - \lambda) d_i^2 \\ 0 \leq \hat{\gamma}_l \leq (1 - \lambda), l \in \mathcal{R}_i \end{cases} \quad (10)$$

where adding (9) and (10) is equivalent to

$$\left\{ \begin{array}{l} \max_{\gamma_l} \sum_{l \in \mathcal{R}_i} h(s_{\max}(l)) \cdot (\bar{\gamma}_l + \hat{\gamma}_l) \\ \text{s.t.} \sum_{l \in \mathcal{R}_i} e(s_{\max}(l)) \cdot L(l) \cdot \bar{\gamma}_l = \lambda d_i^1 \\ \sum_{l \in \mathcal{R}_i} e(s_{\max}(l)) \cdot L(l) \cdot \hat{\gamma}_l = (1 - \lambda) d_i^2 \\ 0 \leq \bar{\gamma}_l \leq \lambda, l \in \mathcal{R}_i \\ 0 \leq \hat{\gamma}_l \leq (1 - \lambda), l \in \mathcal{R}_i \end{array} \right. , \quad (11)$$

which is less or equal to

$$\left\{ \begin{array}{l} \max_{\gamma_l} \sum_{l \in \mathcal{R}_i} h(s_{\max}(l)) \cdot (\bar{\gamma}_l + \hat{\gamma}_l) \\ \text{s.t.} \sum_{l \in \mathcal{R}_i} e(s_{\max}(l)) \cdot L(l) \cdot (\bar{\gamma}_l + \hat{\gamma}_l) = \lambda d_i^1 + (1 - \lambda) d_i^2 \\ 0 \leq \bar{\gamma}_l + \hat{\gamma}_l \leq \lambda + (1 - \lambda), l \in \mathcal{R}_i \end{array} \right. . \quad (12)$$

This equals to

$$\left\{ \begin{array}{l} \max_{\gamma_l} \sum_{l \in \mathcal{R}_i} h(s_{\max}(l)) \cdot \gamma_l \\ \text{s.t.} \sum_{l \in \mathcal{R}_i} e(s_{\max}(l)) \cdot L(l) \cdot \gamma_l = \lambda d_i^1 + (1 - \lambda) d_i^2 \\ 0 \leq \gamma_l \leq 1, l \in \mathcal{R}_i \end{array} \right. \quad (13)$$

$= f_i(\lambda d_i^1 + (1 - \lambda) d_i^2)$. Therefore $\lambda f_i(d_i^1) + (1 - \lambda) f_i(d_i^2) \leq f_i(\lambda d_i^1 + (1 - \lambda) d_i^2)$ and by definition the function f_i is concave.

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