A robust optimization approach to locating and stockpiling marine oil-spill response facilities

Hassan Sarhadi
Fred C. Manning School of Business, Acadia University, Canada
hassan.sarhadi@acadiau.ca

Joe Naoum-Sawaya
Ivey School of Business, Western University, Canada
jnaoum-sawaya@ivey.ca

Manish Verma*
DeGroote School of Business, McMaster University, Canada
mverma@mcmaster.ca

Abstract: In this research, we apply robust optimization to the problem designing emergency response network for marine oil-spills given uncertainty in the location, size and type of the spill, which is collectively defined as oil-spill probability. We consider two robust formulations to model oil-spill probability, i.e., Gamma, and Ellipsoidal, and make use a realistic case study to evaluate the benefits of applying the robust formulations over the one ignoring uncertainty. We show that the higher values of the disutility multiplier play a critical role in the network coverage, and that the more conservative Gamma robustness prescribes substantial initial stockpile thereby obviating the need for further investments whereas Ellipsoidal robustness calls for a gradual investment in response capability thereby enabling resource adjustments for possible future investments.

Keywords: robust optimization; marine oil-spill; emergency response; mixed-integer program; stochasticity.

---

* Corresponding author.

1. Introduction

Globalization and the lowering of trade barriers are largely responsible for the steady growth of international trade over the past decades, which is predominantly conducted over marine networks. Marine, the primary intercontinental mode, transported 1.78 billion tons of crude oil and 1.05 billion tons of refined petroleum products in 2012 (UNCTAD, 2013). It is pertinent that the total volume of oil and petroleum products being carried over marine networks will continue to grow, thanks to the energy needs around the globe. Unfortunately, some of these shipments could result in oil spills and occasional accidents resulting in significant environmental, social, and economic consequences. Two of the most prominent oil-spill related episodes are *Exxon Valdez* in Alaska in 1989 (United States) and then at Prestige in 2002 (Spain), however, the Deepwater Horizon oil-spill of 2010 (United States) is the largest one on record. Although large oil-spills (>30 tonnes) constitute a meagre 0.1% of incidence, they account for almost 60% of the total amount of spillage (Ornitz and Champ, 2002). Given the significance of marine transportation and the adverse impact of oil-spills, this area has attracted a lot of attention from researchers, who have proposed frameworks for oil-spill evaluation, planning and response.

This study focuses on the response planning phase, which entails answering three questions: *first*, where to locate emergency response facilities; *second*, what types of equipment to stockpile at each facility; and *third*, how are the equipment going to be assigned in response to an oil-spill incident. It is important to note that the first two questions deal with the strategic elements of oil-spill response and the related decisions need to be made prior to the oil-spill incidences, whereas the (tactical/operational) dispatching decision is made following the incident. Since the location and capability of the facilities needs to be determined before making the allocation decisions, the setting is comparable to a two-stage facility location problem under uncertainty. In general, and as it will be discussed in the literature review section, there are two ways to solve such problems. One may either assume some stochastic information about the possible outcomes or assume that uncertain information lies within some mathematical structure with no distributional information about it (Snyder 2006). While the former type of problem is suitable for stochastic optimization where the solution seeks to optimize the expected value of the objective function, the latter is more appropriate for robust optimization that looks at the worst-case performance over a set of possible scenarios or intervals of uncertainty. It is pertinent that the ability of robust optimization to effectively handle uncertainties, in the absence of detailed characterization of uncertain elements, make it a favorable approach to deal with optimization problems under uncertainty (Ben-Tal et al., 2009). In addition, robust optimization approach has found numerous applications in diverse contexts such as railway timetabling (Fischetti et al., 2009), portfolio selection (Gregory et al., 2011), inventory management (Bersimas and Thiele, 2006), humanitarian logistics (Ben-Tal et al., 2011), etc. For the oil-spill response problem, accessing precise information on the exact location, size, and type of (infrequent) oil-spill is extremely difficult, and hence the resulting dearth of historical data might undermine the long-term viability of strategic decisions about facilities and equipment. However, such types of problems are amenable to robust optimization. Hence, we will present a two-stage robust program where uncertainty in the three elements associated with an oil-spill, and collectively termed as oil-spill probability will be considered using two different forms of robustness, viz., Gamma and Ellipsoidal.

This paper contributes to the literature by formulating robust models for designing emergency response network for marine oil-spills, when uncertainty in oil-spill probabilities are incorporated via Gamma and Ellipsoidal robustness. We consider a cost-minimizing objective where the emergency response agency must trade off opening response centers and stockpiling sufficient equipment with uncertain (and possibly) high environmental costs in the future. Both the Gamma and Ellipsoidal robust formulations were used to evaluate the fixed versus expected costs, and were tested an approach where uncertainty is ignored, i.e., deterministic or nominal formulation. The three formulations were applied to a realistic case example based in Newfoundland (Canada) and facilitated the following insights: *first*, the inherent trade-off between expected environmental cost versus fixed cost determines the number of facilities to be opened; *second*, higher disutility multiplier values would ensure better coverage for the given marine network, albeit at a higher fixed; *third*, both Gamma and Ellipsoidal formulations yield better coverage than the nominal formulation where uncertainty is ignored, and thereby underscore the significance of robustness for this type of problem; and *fourth*, Gamma robustness is more conservative than Ellipsoidal since it calls for substantial initial investment with little or no need for further investments, whereas the latter prescribes a gradual investment in response capability thereby enabling decision makers to adjust subsequent investments. In sum, to the best our knowledge, this is the first work that applies (Gamma and Ellipsoidal) robust optimization to design emergency response network for marine oil-spills.

The paper is organized as follows. Section 2 provides a detailed critical review of the relevant literature, whereas Section 3 describes the model setting and formulations. Section 4 outlines the customized solution methodology for Ellipsoidal robustness, followed by the description of the realistic case study in Section 5. The performance of the two robust formulations and that of the nominal model in order to demonstrate the expected potential benefits of applying the robust optimization approach to the marine oil-spill response problem are detailed in Section 6, followed by the conclusion and future research directions in Section 7.

2. Literature review

Given the focus of this paper, the relevant papers can be organized under four streams: estimating quantity and cost of oil-spills; oil-spill response; facility location under uncertainty; and, location of emergency response centers with uncertain inputs.

Estimating quantity and cost of oil-spills: Oil-spill modeling is a fairly comprehensive domain that includes discussion about oil type, weather conditions, location, etc., (Reed et al., 1995), most of the relevant contributions fall under two threads: quantity and cost estimation of oil spill. Furthermore, bulk of the work has been done to predict the trajectory and final fate of the oil so as to facilitate appropriate spill response and contingency planning (French-McCay, 2004). However, given the topographical nuances, most of these models were developed in a local context. For instance, Price et al. (2003) focused on the Gulf of Mexico, Aghajanloo et al. (2013) for the Persian Gulf, and Cronk et al. (1990) on Ohio river. On the other hand, estimation of spill related costs has been an active area of research. Etkin (1999) developed the basic (linear) estimates for area-wise cleanup cost, which were later revised to account for cleanup strategy, size of spill, oil type, and shoreline oiling (Etkin, 2000). Subsequently, Vanem et al. (2008) augmented the above works by identifying three main types of damage costs, that is, cleanup, environmental, and socioeconomic. On the other hand, a number of researchers made use of a nonlinear approach to estimate damage costs. For instance, Yamada (2009) proposed a nonlinear regression model between total cost and weight of oil spill, Kontovas et al. (2010) considered other variables to improve the correlation coefficient between the independent and dependent variables, and Psarros et al. (2011) tested the resulting nonlinear models on two separate databases.

Oil-spill response: As noted earlier, oil-spill response is a tactical-level decision and involves prescribing response systems for a specific oil spill, i.e., which types and number of equipment packages to dispatch. Wilhelm and Srinivasa (1997) proposed an integer program and column generation solution method to identify the response, a cutting plane methodology to tackle problems involving medium-to-large oil spills (Srinivasa and Wilhelm, 1997), and an optimization program to develop area-wide contingency planning of oil-spill operations in the Galveston Bay Area (Wilhelm and Srinivasa, 1996). Subsequently, Gkonis et al. (2007) considered the oil weathering process in the oil-spill, and the damage resulting from non-response. More recently, Zhong and You (2011) proposed a multi-objective optimization approach that considers both economic and responsiveness objectives for response operations, while You and Leyffer (2011) proposed a mixed-integer optimization program to minimize the total response cost. To support energy explorations in the remote Arctic areas, Garret et al. (2017) developed a mixed-integer program to model the dynamic network expansion problem which aims to minimize the response time to a set of potential oil spills. Finally, Grubesic et al. (2017) combined simulation with mathematical model to optimize the allocation of response crews and equipment for cleaning up an offshore oil spill in the Gulf of Mexico.

Facility location under uncertainty: There is a considerable body of literature on facility location under uncertainty. Snyder (2006) describes two types of problems: stochastic location problems and robust location problem. A representative work under the former category would be that of Louveaux (1986), who studied a capacitated facility location problem with stochastic information on demands, prices, and costs. On the other hand, Baron et al. (2010) would be a representative work under the latter category. It is pertinent to mention that most of the engagement is in this area has been within the traditional supply chain domain, and we invite the reader to consult Owen and Daskin (1998) and Snyder (2006) for an extensive review of the area, and Gabriel et al. (2014) for the recent advances in robust optimization.

Location of emergency response stations with stochastic inputs: As alluded earlier, the question of locating emergency response facilities has not received much attention. However, we will draw from four notable works dealing with installation of emergency response centers to respond to marine oil spills. Belardo et al. (1984) developed a multi-objective variation of the maximal covering location model of response resources for major maritime oil spills and applied it to a spill in the Long Island Sound (United States). While this model facilitated analyses, its effectiveness was limited because the equipment needs were determined on the basis of a single (standard) spill volume, thereby neglecting the variability in volume of an individual spill, that is, one of the most important features of the oil-spill response problem. Psaraftis et al. (1986) developed a model to determine the locations of the appropriate levels and types of cleanup equipment to respond to oil spills. For each dispatch location, a finite set of response equipment is stockpiled, and the cleanup capability is known. The proposed framework was applied to an illustrative case study in northeastern United States, and insights were provided. Iakovou et al. (1996) proposed an integrated framework that simultaneously considers strategic facility location and tactical equipment allocation decisions. The proposed linear integer program (and the relaxed formulation) was tested on two realistic case studies from Florida (United States). However, this work ignored the probabilistic nature of oil spills, and makes use of historical oil-spill information to make allocation decisions. Finally, Verma et al. (2013) proposed a two-stage stochastic programming model to locate emergency response facilities and stockpile equipment in the first stage and make allocation decision in the second stage. Publicly available information was used to build a case study in Newfoundland (Canada), where oil-spill probabilities were estimated from historical annual spill frequency data. As alluded earlier, the infrequent oil-spill incidences and the resulting dearth of historical data could undermine the long-term viability of strategic decisions about facilities and equipment. Hence, our study is motivated by the inherent uncertainty surrounding spill probability data, and thus is motivated by Verma et al. (2013).

To the best of our knowledge, no previous work models location of oil-spill response facilities and equipment stockpile decisions through robust optimization framework using two forms of robustness – i.e., Gamma and Ellipsoidal, which we intend to do in this paper. The inherent uncertainty around spill probability make the problem amenable to robust optimization.

3. Marine oil-spill response planning program

For expositional reasons, in this section, we first present the nominal formulation of the marine oil-spill response program as introduced in Verma et al. (2013), and then develop the robust versions.

3.1 Nominal formulation: Two-stage stochastic program with recourse

The oil-spill response planning program necessitates making decisions about locating facilities and acquiring equipment packages in the first stage, whereas the recourse component considers information about oil-spill incidents to make equipment response and dispatch decisions. For brevity, we represent the two-stage stochastic programming problem with recourse as a single optimization program (Ruszczynski and Shapiro, 2003; Sen and Higle, 1999). However, we first introduce the variables and parameters.

SETS

I: set of possible facility sites, indexed by i;
E: set of equipment packages, indexed by e;
J: set of oil-spill zones, indexed by j;
K: set of oil-spill profiles, indexed by k;
$I_{jk}^e = \{i \in I | t_{ij}^e < T_{jk}\}$: set of sites i such that an equipment package of type e can be dispatched soon enough to contain a spill of type k in zone j

PARAMETERS

F_i: fixed cost to open facility at site i;
A_i^e: acquisition cost to acquire equipment package e at site i;
EC_{jk}: environmental cost resulting from non-containment of oil-spill type k in zone j;
OC_{jk}^e: cost to operate one unit of equipment package e in zone j for oil-spill type k;
TC_{ij}^e: cost to transport one unit of equipment package e from site i to zone j;
v_{jk}: volume of oil-spill to be contained of spill type k in zone j;
p_{jk}: probability of oil-spill of type k in zone j;
t_{ij}^e: travel time for equipment package e from site i to zone j;
T_{jk}: cut-off time to respond to an oil-spill of type k in zone j;
C_{jk}^e: maximum amount of oil that an equipment package e can contain for oil-spill type k in zone j

DECISION VARIABLES

$$Y_i = \begin{cases} 1 \text{ if facility at site } i \text{ is opened,} \\ \quad 0 \text{ otherwise;} \end{cases}$$
U_i^e: number of equipment packages of type e at site i;
Z_{jk}: fraction of oil-spill of type k contained in zone j;

N^e_{ijk}:   number of equipment package of type e dispatched from site i for oil-spill of type k in zone j.

Let random variable s represent a given oil-spill incident scenario jointly defined by the spill-location $J(s)$, and spill-volume $K(s)$. Hence, the total number of scenarios is $|S|=|J|*|K|$. Now combining the two stages of the stochastic program yields the single-level model *viz.,* (SLM).

(SLM)

Minimize

$$\sum_{i\in I} F_i Y_i + \sum_{i\in I}\sum_{e\in E} A^e_i U^e_i + \sum_{j\in J}\sum_{k\in K} p_{jk}\left[EC_{jk}(1-Z_{jk}) + \sum_{i\in I}\sum_{e\in E}(OC^e_{jk} + TC^e_{ij})N^e_{ijk}\right] \quad (1)$$

Subject to:

$$MY_i - U^e_i \geq 0 \qquad \forall i \in I, \forall e \in E \qquad (2)$$
$$\sum_{i\in I^e_{jk}} N^e_{ijk} \leq U^e_i \qquad \forall i \in I, \forall e \in E \qquad (3)$$
$$\sum_{i\in I^e_{jk}} \sum_{e\in E} C^e_{jk} N^e_{ijk} \geq v_{jk} Z_{jk} \qquad \forall j \in J, \forall k \in K \qquad (4)$$
$$Y_i \in \{0,1\} \qquad \forall i \in I \qquad (5)$$
$$U^e_i \geq 0 \text{ integer} \qquad \forall i \in I, \forall e \in E \qquad (6)$$
$$N^e_{ijk} \geq 0 \text{ integer} \qquad \forall i \in I, \forall j \in J, \forall k \in K, \forall e \in E \qquad (7)$$
$$0 \leq Z_{jk} \leq 1 \qquad \forall j \in J, \forall k \in K \qquad (8)$$

The objective function represented by (1) contains the facility location and equipment acquisition decisions, plus the expected cost of oil spills and dispatch decisions. Constraint set (2) specifies that no equipment packages can be acquired unless the corresponding facilities are open. Constraint set (3) ensures that the total number of equipment packages dispatched from any facility does not exceed the number of corresponding packages acquired at that facility. Constraint set (4) ensures that the total number of equipment packages of a specific type dispatched in response to a particular oil-spill incident is at least as large as the requirement. Finally, constraint sets (5) to (8) specify the ranges of the variables.

3.2 Robust formulations

It is relevant that Verma et al. (2013) made use of historical data to develop probability estimates for the location and type of oil-spills, i.e., p_{jk}, and hence the resulting analyses was contingent on the data. In an effort to facilitate improved decision-making, we next outline robust formulations of the same oil-spill response planning problem. As alluded, robust optimization does not make any assumptions about the probability distribution of the uncertain parameters, but only assumes that the uncertain parameter resides in the so called “uncertainty set”. More precisely, the uncertainty set contains a range of values for the uncertain parameters that will be considered in the robust optimization program. Although a wide variety of uncertainty sets have been introduced in the literature, we use the most popular ones viz., Gamma (Bertsimas and Sim, 2004) and Ellipsoidal uncertainty (Ben-Tal et al., 2009) for the proposed managerial problem.

3.2.1 Gamma robustness

Gamma robustness has been developed by Bertsimas and Sim (2004) to model uncertain parameters. For the proposed problem p_{jk} is the uncertain parameter, which varies within the interval $[\bar{p}_{jk}, \bar{p}_{jk} + \Delta p_{jk}]$ where $\bar{p}_{jk}$ is the nominal value of oil-spill probability of type *k* in zone *j* and that is obtained from historical data, and Δp_{jk} is a parameter that describes the maximum deviation from the nominal probability. In addition, we assume that a finite number of oil-spill location & type pairs can deviate from their nominal probability, also known as uncertainty budget. We next introduce some additional notations, and then develop the robust version of the program.

$\Xi = \{(j,k)|\Delta p_{jk} > 0\}$: set of pairs with positive deviation from their nominal probability;

L: uncertainty budget or the maximum number of pairs with positive deviation from their nominal probability.

The objective function of the robust response with Gamma robustness (RRwG) is:

$$min\Big\{\sum_{i\in I} F_i Y_i + \sum_{i\in I}\sum_{e\in E} A_i^e U_i^e + \sum_{j\in J}\sum_{k\in K} \bar{p}_{jk}\left[EC_{jk}(1-Z_{jk}) + \sum_{i\in I}\sum_{e\in E}(OC_{jk}^e + TC_{ij}^e)N_{ijk}^e\right] + \max_{\xi\subseteq\Xi,|\xi|\le L,\forall(j,k)\in\Xi}\left[\sum_{(j,k)\in\xi}\Delta p_{jk}\left[EC_{jk}(1-Z_{jk}) + \sum_i\sum_e(OC_{jk}^e + TC_{ij}^e)N_{ijk}^e\right]\right]\Big\} \quad (9)$$

(9) minimizes the sum of the cost of the nominal problem and the maximum cost imposed by the uncertain parameter. The maximization component can be simplified by introducing a variable MO_{jk} for each pair (j,k):

$$max\ \left[\sum_{(j,k)\in\xi}\Delta p_{jk}\left[EC_{jk}(1-Z_{jk}) + \sum_i\sum_e(OC_{jk}^e + TC_{ij}^e)N_{ijk}^e\right]MO_{jk}\right] \quad (10)$$

subject to:

$$\sum_{(j,k)\in\xi} MO_{jk} \le L \quad (11)$$

$$0 \le MO_{jk} \le 1, \forall(j,k)\in\xi \quad (12)$$

The dual of the maximization problem, i.e., (10)-(12), for a given value of variables Z_{jk} and N_{ijk}^e such as ($Z_{jk}^*, N_{ijk}^{e,*}$) will then be:

$$min\left[\sum_j\sum_k \rho_{jk} + L\varphi\right] \quad (13)$$

subject to:

$$\varphi + \rho_{jk} \ge \Delta p_{jk}\left[EC_{jk}(1-Z_{jk}^*) + \sum_i\sum_e(OC_{jk}^e + TC_{ij}^e)N_{ijk}^{e,*}\right] \quad \forall j\in J, \forall k\in K \quad (14)$$

$$\varphi \ge 0 \quad (15)$$

$$\rho_{jk} \ge 0 \quad \forall j\in J, \forall k\in K \quad (16)$$

where, φ is the dual variable associated with constraint set (11), and ρ_{jk} with constrain set (12). Finally, (13)-(16) can be combined with the first component of (9) to yield **(RRwG)**:

$$min\left[\sum_i F_i Y_i + \sum_i\sum_e A_i^e U_i^e + \sum_j\sum_k \bar{p}_{jk}\left[EC_{jk}(1-Z_{jk}) + \sum_i\sum_e(OC_{jk}^e + TC_{ij}^e)N_{ijk}^e\right] + \sum_j\sum_k \rho_{jk} + L\varphi\right] \quad (17)$$

subject to:

$$\varphi + \rho_{jk} \ge \Delta p_{jk}\left[EC_{jk}(1-Z_{jk}) + \sum_i\sum_e(OC_{jk}^e + TC_{ij}^e)N_{ijk}^e\right] \quad \forall j\in J, \forall k\in K \quad (18)$$

(2)-(8), (15)-(16).

3.2.2 Ellipsoidal robustness

Ellipsoidal robustness, developed by Ben-Tal et al. (2009), assumes that the uncertain parameter resides within an interval. More specifically, for the proposed planning program, it will allow the probability of oil-spill p_{jk} to deviate from its nominal value $\bar{p}_{jk}$ utmost by Δp_{jk} such that $p_{jk} \in [\bar{p}_{jk} - \Delta p_{jk}, \bar{p}_{jk} + \Delta p_{jk}]$. Note that although the deviation from the nominal probability Δp_{jk} for a given (j, k) is unknown, we assume that the total sum of squared deviations from the nominal values is capped, and is within an Ellipsoid:

$$\sqrt{\sum_j \sum_k \Delta p_{jk}^2} \leq \beta \tag{19}$$

where, β is the parameter that controls the level of uncertainty. The objective function of the robust version of the planning program can then be cast as follows:

$$min\Big\{\sum_i F_i Y_i + \sum_i \sum_e A_i^e U_i^e + \sum_j \sum_k \bar{p}_{jk} \left[EC_{jk}(1 - Z_{jk}) + \sum_i \sum_e (OC_{jk}^e + TC_{ij}^e) N_{ijk}^e\right] + \\ max\left[\sum_j \sum_k \Delta p_{jk} \left[EC_{jk}(1 - Z_{jk}) + \sum_i \sum_e (OC_{jk}^e + TC_{ij}^e) N_{ijk}^e\right]\right]\Big\} \tag{20}$$

The maximization component of (20) can be rewritten as:

$$max\{\sum_j \sum_k \Delta p_{jk} \left[EC_{jk}(1 - Z_{jk})\right]\} + max\{\sum_j \sum_k \Delta p_{jk} (\sum_i \sum_e (OC_{jk}^e + TC_{ij}^e) N_{ijk}^e)\} \tag{21}$$

Each of the two maximization components can be further expanded. For the first maximization, we have:

$$max\{\sum_j \sum_k \Delta p_{jk} \left[EC_{jk}(1 - Z_{jk})\right]\} = max[\sum_j \sum_k \Delta p_{jk} EC_{jk}] - max[\sum_j \sum_k \Delta p_{jk} EC_{jk} Z_{jk}] \tag{22}$$

Using Cauchy-Scwharz inequality on each of the maximization terms in (22), we have:

$$max[\sum_j \sum_k \Delta p_{jk} EC_{jk}] \to \|\Delta p_{jk}\| \times \|EC_{jk}\| \to \beta \times \sqrt{\sum_j \sum_k EC_{jk}^2} \tag{23}$$

$$max[\sum_j \sum_k \Delta p_{jk} EC_{jk} Z_{jk}] \to \|\Delta p_{jk}\| \times \|EC_{jk}\| \times \|Z_{jk}\| \to \beta \times \sqrt{\sum_j \sum_k EC_{jk}^2} \times \sqrt{\sum_j \sum_k Z_{jk}^2} \to \beta\gamma \tag{24}$$

where,

$$\gamma = \sqrt{\sum_j \sum_k EC_{jk}^2} \sqrt{\sum_j \sum_k Z_{jk}^2} \tag{25}$$

For the second maximization in (21), we introduce:

$$\theta_{ijk}^e = OC_{jk}^e + TC_{ij}^e \tag{26}$$

then we have:

$$\max[\sum_j \sum_k \Delta p_{jk} \sum_i \sum_e (OC_{jk}^e + TC_{ij}^e) N_{ijk}^e] \to max[\sum_j \sum_k \Delta p_{jk} (\theta_{1jk}^1 N_{1jk}^1 + \theta_{2jk}^1 N_{2jk}^1 + \theta_{3jk}^1 N_{3jk}^1 + \\ \dots)] \to max[\sum_j \sum_k \Delta p_{jk} \theta_{1jk}^1 N_{1jk}^1 + \sum_j \sum_k \Delta p_{jk} \theta_{2jk}^1 N_{2jk}^1 + \cdots] \tag{27}$$

Using Cauchy-Scwharz inequality for each term in (27), we have:

$$\|\Delta p_{jk}\| \times \|\theta^1_{1jk} N^1_{1jk}\| + \|\Delta p_{jk}\| \times \|\theta^1_{2j}\ \ N^1_{2jk}\| + \|\Delta p_{jk}\| \times \|\theta^1_{3jk} N^1_{3j}\ \ \| + \cdots \rightarrow \beta \times \left(\sqrt{\sum_j \sum_k {\theta^1_{1jk}}^2 {N^1_{1jk}}^2} + \sqrt{\sum_j \sum_k {\theta^2_{1jk}}^2 {N^2_{1j}}^2} + \sqrt{\sum_j \sum_k {\theta^3_{1jk}}^2 {N^3_{1jk}}^2} + \cdots\right) \rightarrow \beta(\alpha^1_1 + \alpha^2_1 + \alpha^3_1 + \ldots) \rightarrow \beta \sum_i \sum_e \alpha^e_i$$

When α^e_i satisfies the following constraint:

$$ {\alpha^e_i}^2 - \sum_j \sum_k {\theta^e_{ijk}}^2 {N^e_{ijk}}^2 \geq 0$$

The robust response with Ellipsoidal robustness (**RRwE**) formulation then yields:

$$min\left[\sum_i F_i Y_i + \sum_i \sum_e A^e_i U^e_i + \sum_j \sum_k \bar{p}_{jk} \left[EC_{jk}\left(1 - Z_{jk}\right) + \sum_i \sum_e \left(OC^e_{jk} + TC^e_{ij}\right) N^e_{ijk}\right] + \beta\gamma + \sum_i \sum_e \beta\, \alpha^e_i\right] \tag{28}$$

subject to:

$$\gamma^2 - (\sum_j \sum_k EC^2_{jk})(\sum_j \sum_k Z^2_{jk}) \geq 0 \tag{29}$$

$${\alpha^e_i}^2 - \sum_j \sum_k {\theta^i_{ejk}}^2 {N^e_{ijk}}^2 \geq 0 \qquad \forall i \in I, \forall e \in E \tag{30}$$

(2)-(8), (26)

<u>Alternative formulation of (RRwE):</u>

Note that (29) and (30) are second-order cone constraints, which can hinder the computational performance of the model. Thus, we introduce two new variables to reduce the number of second-order cone constraints thereby improving the performance of the model. More specifically in (21), we replace $\left(1 - Z_{jk}\right)$ by ω_{jk}, and $\sum_i \sum_e \left(OC^e_{jk} + TC^e_{ij}\right) N^e_{ijk}$ by μ_{jk}, to yield:

$$max\left(\sum_j \sum_k \Delta p_{jk} \left[EC_{jk}\omega_{jk}\right]\right) + max\left(\sum_j \sum_k \Delta p_{jk}\mu_{jk}\right) \tag{31}$$

Using Cauchy-Scwharz inequality for each maximization subproblem in (31) we have:

$$max\left[\sum_j \sum_k \Delta p_{jk} \left[EC_{jk}\left(\omega_{jk}\right)\right]\right] = \beta \times \sqrt{\sum_j \sum_k EC^2_{jk}\omega^2_{jk}} \rightarrow \beta\lambda,\ \lambda = \sqrt{\sum_j \sum_k EC^2_{jk}\omega^2_{jk}}$$

$$max\left[\sum_j \sum_k \Delta p_{jk}\, \mu_{jk}\right] = \beta \times \sqrt{\sum_j \sum_k \mu^2_{jk}} \rightarrow \beta\pi,\ \pi = \sqrt{\sum_j \sum_k \mu^2_{jk}}$$

Therefore, the Reformulated RRwE model (**RRRwE**) becomes:

$$min\left[\sum_i F_i Y_i + \sum_i \sum_e A^e_i U^e_i + \sum_j \sum_k \bar{p}_{jk} \left[EC_{jk}\omega_{jk} + \sum_i \sum_e \left(OC_{jk} + TC_{ij}\right) N^e_{ijk}\right] + (\beta\lambda) + (\beta\pi)\right] \tag{32}$$

Subject to:

(2)-(3), (5)-(8)

$$\lambda^2 - \left(\sum_j \sum_k EC^2_{jk}\omega^2_{jk}\right) \geq 0 \tag{33}$$

$$\pi^2 - \sum_j \sum_k \mu^2_{jk} \geq 0 \tag{34}$$

$$\mu_{jk} \geq \sum_i \sum_e \left(OC^e_{jk} + TC^e_{ij}\right) N^e_{ijk} \qquad \forall j \in J, \forall k \in K \tag{35}$$

$$\sum_{i \in I^e_{jk}} \sum_{e \in E} C^e_{jk} N^e_{ijk} \geq v_{jk}\left(1 - \omega_{jk}\right) \qquad \forall j \in J, \forall k \in K \tag{36}$$

$\omega_{jk} + Z_{jk} \geq 1$ $\quad \forall j \in J, \forall k \in K \quad$ (37)

$1 \geq \omega_{jk} \geq 0$ $\quad \forall j \in J, \forall k \in K \quad$ (38)

4. Solution Methodology for Ellipsoidal robust formulations

Introducing new variables in (21) did manage to reduce the number of second-order conic constraints, however, the number of second-order cone constraints to be considered in **RRwE** is still large to pose computational challenge. To alleviate the challenge, we outline a cutting plane approach aimed both at reducing the number of times the second-order cone constraints are called and at improving the overall computational performance. To this end, at each iteration of the cutting plane approach, a Master Problem (MP) and a Sub Problem (SP) will be called, whose details are outlined next.

Master Problem (MP)

(MP) is a simpler version of **RRwE** since it does not have the second-order cone constraints, and instead has cutting planes that estimate the cost imposed by uncertain parameters. We introduce *h* as the iteration index of the cutting plane, which yields (MP) as follows:

$$min\left[\sum_i F_i\, Y_i + \sum_i \sum_e A_i^e\, U_i^e + \sum_j \sum_k \bar{p}_{jk}\left[EC_{jk}\left(1 - Z_{jk}\right) + \sum_i \sum_e \left(OC_{jk}^e + TC_{ij}^e\right) N_{ijk}^e\right] + \theta\right] \quad (39)$$

subject to:

$$\theta \geq \sum_j \sum_k \Delta p_{jk}^h \left[EC_{jk}\left(1 - Z_{jk}\right) + \sum_i \sum_e \left(OC_{jk}^e + TC_{ij}^e\right) N_{ijk}^e\right] \quad (40)$$

(2)-(8)

Note that (40) generates cutting plane at each iteration *h*, and that (MP) iteratively maximizes the cost imposed by the worst-case oil-spill by accumulating cutting planes generated in each iteration. In order to create cuts at each iteration *h*, the value Δp_{jk}^h is supplied by (SP). More precisely, in (MP), the value of Δp_{jk}^h is fixed to its optimal value obtained by solving (SP) to optimality in iteration *h-1*.

Sub Problem (SP)

$$max\left[\sum_i F_i\, \bar{Y}_i + \sum_i \sum_e A_i^e\, \bar{U}_i^e + \sum_j \sum_k \bar{p}_{jk}\left[EC_{jk}\left(1 - \bar{Z}_{jk}\right) + \sum_i \sum_e \left(OC_{jk}^e + TC_{ij}^e\right) \bar{N}_{ijk}^e\right] + \alpha\right]$$

subject to:

$$\sqrt{\sum_j \sum_k (\Delta p_{jk}^{h+1})^2} \leq \beta$$

$$\sum_j \sum_k \Delta p_{jk}^{h+1}\left[EC_{jk}\left(1 - \bar{Z}_{jk}\right) + \sum_i \sum_e \left(OC_{jk}^e + TC_{ij}^e\right) \bar{N}_{ijk}^e\right] \geq \alpha$$

$$\Delta p_{jk}^{h+1} \geq 0$$

In each iteration h of the cutting plane procedure, (SP) finds the optimal value of the change in probability for the next iteration, i.e., Δp_{jk}^{h+1}, which is then passed to (MP) in the next iteration. Note that $\bar{Y}_i$, $\bar{U}_i^e$, $\bar{Z}_{jk}$, and $\bar{N}_{ijk}^e$ are constants in (SP), and that their values are gained by solving MP to optimality. Figure 1 presents the schematic about the (MP) and (SP) interaction the cutting plane procedure.

While objective functions of (MP) and (SP) become close enough, **Do**

Step1: **(**MP) is solved to optimality while Δp_{jk}^{h} is fixed to the optimal values obtained from (SP) in the previous iteration. To begin with the procedure, in the first iteration (i.e., $h = 1$) all Δp_{jk}^{h} variables are set to zero.

Step 2: Once (MP) is solved, the optimal values of its decision variables (i.e., $\bar{Y}_i^h$, $\bar{U}_i^{e,h}$, $\bar{Z}_{jk}^h$, and $\bar{N}_{ijk}^{e,h}$) are sent to (SP) to constitute its objective function and constraints.

Step 3: Iteration is incremented, **(**SP) is solved to optimality and the optimal value of its decision variable (i.e., Δp_{jk}^{h+1}) is sent to (MP). Then the iteration index is incremented ($h \leftarrow h + 1$) and the procedure returns to Step 1.

End

Figure 1: Interaction of (MP) and (SP) within cutting plane procedure

Generating cuts through Branch-and-Cut approach: The cutting plane procedure outlined in Figure 1 requires repeatedly solving (MP) as a mixed-integer program, and (SP) with second-order cone constraints to generate violated cuts. To boost the performance, the cuts can be added through a branch-and-cut procedure such that the violated cuts are generated in each node of the branch-and-bound tree with an integer incumbent solution. The branch-and-cut procedure can be described in two main steps:

1. Start with a relaxation of RRRwE or RRwE where the second-order cone constraints, i.e., (33)-(34) or (29)-(30), are dropped.
2. Solve the relaxed problem through the branch-and-bound approach and at each node of the tree with an integer solution check if the second-order cone constraints are violated. If so, reject the current incumbent solution, create the corresponding cut and add it to the relaxed problem.

It is pertinent to add that under the branch-and-cut procedure, at the outset, both the integrality and second-order cone constraints are relaxed. Subsequently, the integrality constraints are imposed through branching while the second-order cone constraints are enforced by generating cuts and adding the violated ones to each node of the tree with an incumbent integer solution.

5. Case study

In an effort to demonstrate the value of robustness, we revisit the managerial problem revealed via reports developed by Transport Canada (2007, 2010), which formed the basis of the case study analyzed in Verma et al. (2013). For expositional reasons, and to provide enough details, we are reproducing the relevant input data, and acknowledge the appropriate sources. Figure 2 depicts the area of interest (AOI) along the south coast of Newfoundland, which stretches from Port aux Basques to St. John's (Transport

Canada, 2007). It is important to note that the AOI, roughly 50 nautical miles wide along the southern coastline, accommodates over 20,000 vessel movements annually. However, given the heterogeneous attributes, the AOI has been divided into five zones: zones 1 and 2 collectively comprise the Placentia Bay, zones 3 and 4 provide transit to passage to the vessels, while zone 5 surrounds the provincial capital of St. John's. It is interesting that just outside zone 5, approximately 250K liters of crude oil spilled on 17 November 2018 (CBC, 2018), which further underscores the significance of this research. Note that given the varying level of activity each zone is a likely location for oil (or fuel) spill, whose profile would be determined by oil type, weather conditions, and volume spilled. Finally, while we have attempted to capture all of the pertinent factors, our work does not consider issues such as weathering and movement of oil slick, time-dependent oil physiochemical properties, etc.

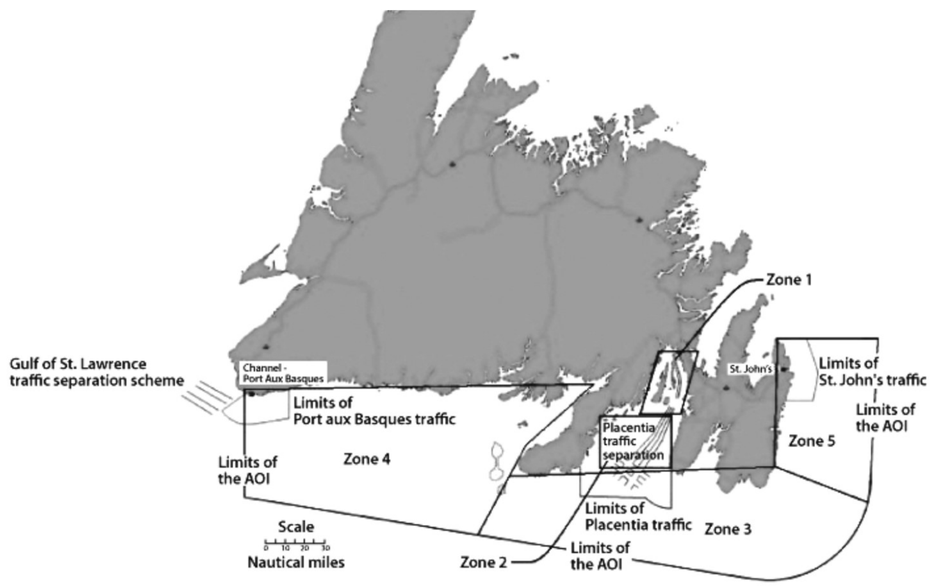

FIGURE 2: Area of Interest (Source: Transport Canada, 2007)

Parameter estimation: Transport Canada (2007, 2010) considered over 1000 accident records from 1980 to 2005, and the projected marine activities in the region, to propose the annual spill frequencies in the five zones for the three relevant types of oil, i.e., crude oil, refined products, and fuel oil. However, we convert the annual frequencies into probabilities, while ensuring that the original spill size categorization expressed in barrels is respected. Though the report specifies six spill sizes – i.e., 1 to 49; 50 to 999; 1000 to 9999; 10,000 to 99,999; 100,000 to 199,999; and, greater than 200,000, however for expositional ease, we use the legends 1st, 2nd, 3rd, 4th, 5th and 6th to refer to the six ranges, respectively. Table 1 provides the requisite probabilities.

Transport Canada (2007) also provides an estimate of environmental cost associated with the six spill ranges based on oil-viscosity. The latter is relevant since medium and high-persistence oil, correspond to the dissipation characteristics of "fuel & refined" and "crude oil", respectively. Table 2 reports the values for all spill sizes, and zones. Finally, the oil-spill cleanup cost for different volumes was estimated using the spill-cost model proposed in Etkin (1999).

Oil type/ spill range	Zone	1st	2nd	3rd	4th	5th	6th
Crude	**1**	1.26E-01	1.59E-02	4.99E-03	1.15E-03	1.72E-04	5.94E-04
Fuel		9.26E-03	2.51E-02	1.32E-03	-	-	-
Refined		9.84E-02	2.12E-02	4.02E-03	1.53E-04	1.35E-04	6.85E-05
Crude	**2**	2.92E-02	3.69E-03	6.60E-04	6.60E-04	5.91E-05	2.20E-04
Fuel		2.65E-02	7.67E-02	2.65E-03	-	-	-
Refined		2.42E-02	5.23E-03	9.13E-04	9.79E-05	5.16E-05	2.51E-05
Crude	**3**	3.68E-02	4.64E-03	8.31E-04	8.31E-04	7.45E-05	2.75E-04
Fuel		2.51E-02	7.28E-02	2.65E-03	-	-	-
Refined		3.52E-02	7.59E-03	1.32E-03	1.43E-04	7.71E-05	3.57E-05
Crude	**4**	7.58E-03	9.55E-04	1.71E-04	1.71E-04	1.59E-05	5.69E-05
Fuel		8.47E-02	2.42E-02	7.94E-03	-	-	-
Refined		9.50E-03	2.05E-03	3.57E-04	3.84E-05	1.59E-05	9.26E-06
Crude	**5**	2.47E-04	3.18E-06	5.29E-06	5.29E-06	5.29E-07	2.78E-05
Fuel		4.50E-02	1.30E-01	3.97E-03	-	-	-
Refined		1.20E-02	2.59E-03	4.79E-04	2.78E-05	1.85E-06	1.85E-06

Table 1: Probability (p_{jk})of oil-spill in the area of interest

Oil type	Zone	1st	2nd	3rd	4th	5th	6th
Fuel & Refined	**1**	90	1568	10,740	89,435	179,082	349,135
	2	166	1676	11,118	94,479	184,897	350,664
	3	111	1237	8195	67,337	134,795	261,464
	4	95	1574	10,748	89,430	179,123	484,713
	5	64	810	5444	47,003	93,309	175,557
Crude	**1**	140	2389	16,356	136,812	278,726	529,426
	2	322	2658	17,320	142,010	280,147	532,513
	3	166	1901	12,540	102,485	209,907	401,408
	4	147	2395	16,357	135,959	308,618	665,029
	5	102	1240	10,425	70,976	141,595	266,612

Table 2: Environmental cost (EC_{jk}) ($ thousands)

Candidate sites for locating oil-spill response facilities: The topography of Newfoundland, coupled with (marine and road) accessibility and physical installation space constrained eight potential sites for locating oil-spill response facilities. Figure 3, borrowed from Verma et al. (2013), depicts the seven sites while Port aux Basques can be located in Figure 2. Table 3 provides the cost and coverage area for each of the eight candidate sites within the predefined critical time period. It is pertinent that the fixed cost is not the same for all the locations, and were arrived at based on personal communication with the Canadian

Coast Guard (Verma et al., 2013). Finally, a zone is deemed covered only if every point in that zone can be reached within 6 hours.

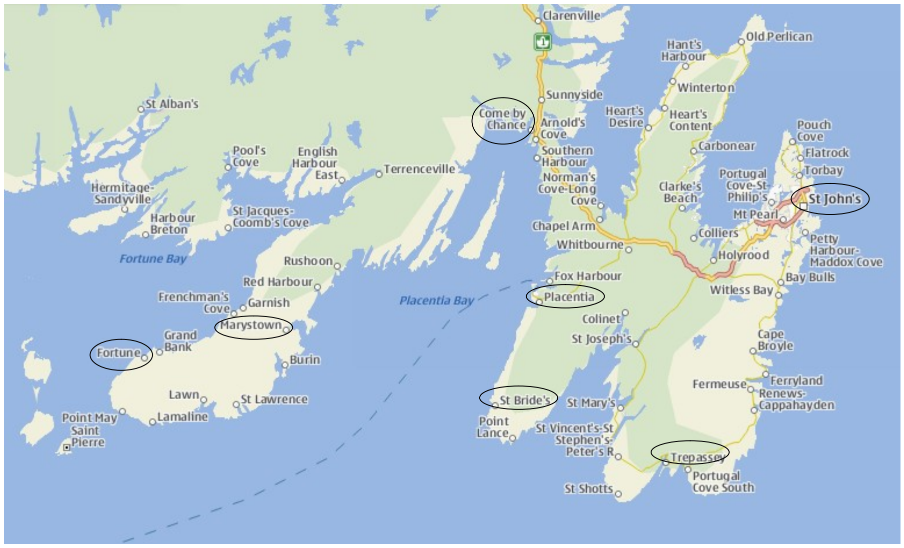

Figure 3: Potential Facility Locations (Source: Verma et al., 2013)

Candidate Sites	Fixed Cost ($ millions)	Zone coverage				
		1	2	3	4	5
St. John's (StJ)	2.0	√	√	√		√
Whiffen Head (Whd)	2.0	√	√			
Placentia Bay (Pla)	3.0	√	√			
Mary's Town (MTn)	3.0	√	√			
Port of Basque (PoB)	3.0				√	
Fortune (For)	4.0		√	√	√	
Saint Bride's (StB)	4.0	√	√	√		
Trepassey (Tre)	4.0			√		√

Table 3: Facility cost (F_i)and coverage areas

Equipment related costs: For each oil-type, we considered six types of equipment to match the oil-spill profile for a total of 18 different equipment packages. More precisely, for each oil-type, we consider cleanup capacities up to 50 tons; 100 tons; 500 tons; 1000 tons; 5000 tons; and, 10,000 tons. The acquisition costs of these equipment are borrowed from Verma et al. (2013), whereas the operating and transport cost have been approximated from the rate schedule of East Coast Response Center (ECRC, 2005) and as they appear in Verma et al. (2013). The three costs for the six equipment types are reproduced in Table 4.

Spill volumes: As alluded earlier, Transport Canada organizes spill volumes into six ranges. We take the mid-point of each range as the spill volume, and note that any other specific value from the range can similarly be input to simulate the resulting impact. Thus, we assume spill-volume values of 25 tons; 500 tons; 5000 tons; 50,000 tons; 150,000 tons; and, 200,000 tons.

Cost	Volume of crude oil (tons)					
	50	**100**	**500**	**1000**	**5000**	**10000**
Equipment acquisition	450	900	4500	9000	45,000	90,000
Operating	25	39	151	302	1444	2888
Transport	10	19	90	190	900	1900

Table 4: Equipment related costs ($ thousands)

6. Computational results

In this section, we first provide the results of solving the oil-spill response problem using the two types of robustness followed by the relevant analysis, which in turn enables us to comment on the value of incorporating robustness, and conclude with a brief comment on the computational performance.

6.1 Ellipsoidal robustness solution

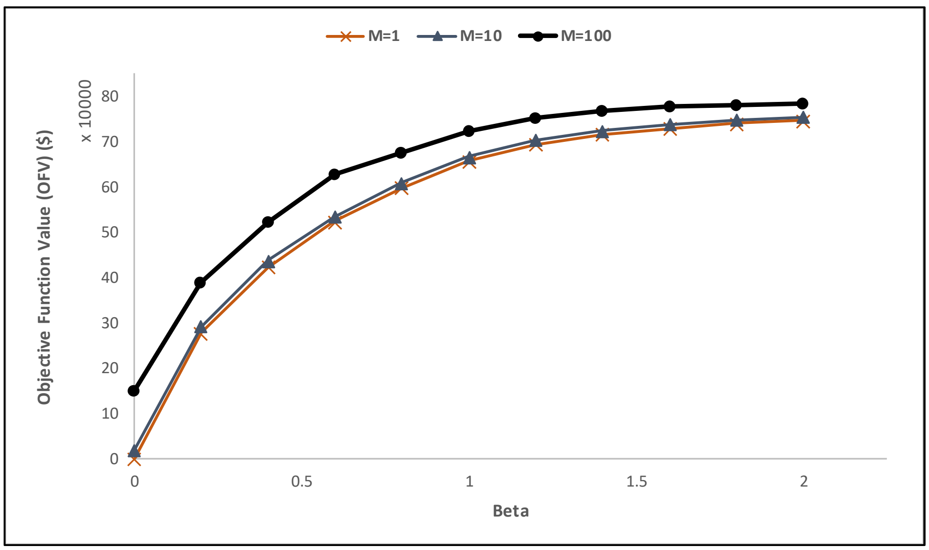

Figure 4: Impact of disutility multipliers and beta on OFV

For Ellipsoidal robustness, it is important to understand the impact of the disutility multiplier and the radius of uncertainty (measured as beta) on the objective function value (OFV). Note that disutility multipliers are intentioned to scale up the miniscule probabilities of oil-spills and facilitate comparison between various costs in the two stages of **(RRwE)**, whereas beta, as expressed in equation (19), is a measure of uncertainty. Figure 4 depicts the impact of the two elements on the OFV, and enables us to make the following three deductions: *first*, for a given value of the disutility multiplier, larger values of beta results in higher OFV to account for increased uncertainty; *second*, for a given value of beta, larger values of the disutility multiplier implies more emphasis on the 2nd stage cost that in turn increases the OFV; and *third*, all the three OFV curves are concave, i.e., for a given disutility multiplier value, OFV increases with the increase in beta at a decreasing rate and then stabilizes.

Beta Values	M=1		M=10		M=100	
	1st stage	**2nd stage** *Env.* *Disp.* *Wcc.*	**1st stage**	**2nd stage** *Env.* *Disp.* *Wcc.*	**1st stage**	**2nd stage** *Env.* *Disp.* *Wcc.*
0.0	0	*2101* *0* *0*	0	*21012* *0* *0*	59000	*86988* *4932* *0*
0.2	96350	*1568* *0.86* *180326*	99500	*15381* *9* *7057*	140000	*7634* *4955* *235754*
0.4	193550	*1431* *1.24* *229044*	194450	*14255* *12.5* *228186*	255650	*59503* *5122* *203305*
0.6	264650	*1329* *2* *256910*	269600	*13079* *23* *252090*	337100	*46614* *5291* *237598*
0.8	313150	*1211* *3.5* *274464*	327200	*11991* *37.5* *270455*	384800	*39518* *5395* *245521*
1.0	423050	*1092* *5.2* *233329*	426200	*10858* *54* *230219*	499100	*30346* *5535* *189491*
1.2	529700	*1014* *6.4* *162779*	53060	*10111* *64* *161899*	578750	*25525* *5603* *143479*
1.4	598550	*976* *6.9* *116064*	602600	*9556* *74* *112112*	669650	*20206* *5699* *73661*
1.6	637700	*921* *8.5* *90631*	638150	*9129* *83* *90214*	712850	*14500* *5812* *44240*
1.8	652100	*897* *8.7* *86686*	669650	*8297* *99* *69417*	738950	*10769* *5895* *25494*
2.0	987650	*785* *10.8* *58378*	692150	*7560* *112* *53997*	746600	*9380* *5927* *21848*

Table 5: Components of OFV ($ ’000) using Ellipsoidal robustness

Table 5 lists the different components of OFV and could be used to analyze the behavior in Figure 4. For each of the three disutility multiplier values, and for each value of beta, the OFV has been decomposed into the 1st stage and the 2nd stage costs, where the latter contains three elements: environment cost (*Env.*); dispatching cost (*Disp.*); and, worst case cost (*Wcc*). It is clear that the 1st stage cost increases with beta, which in turn indicates that increased uncertainty consideration (i.e., higher beta) necessitates spending more money to install better safeguard (i.e., facility and equipment stockpile) against oil-spills. Doing so, however, results in an increase in dispatching cost since more equipment need to be transported to the spill location, and a decrease in environment cost because of higher capability to respond to oil spills. It is pertinent to note that the cost imposed by the worst-case exhibits an increasing trend, and then starts decreasing for $\beta \geq 1$. This we believe is the reason behind the stabilizing of the OFV in Figure 4. Finally, it is worthwhile examining the results when $\beta = 0$, i.e., equivalent to the deterministic setting or nominal formulation of Subsection 3.1. For disutility multiplier values of 1 and 10, the model decides to not open any facility and simply incur the (relatively miniscule) environment cost. However, a value of 100 magnified the role of the 2nd stage costs and in turn forced the model to open facilities and acquire equipment so that exorbitant costs associated with no response be avoided.

6.2 Gamma robustness solution

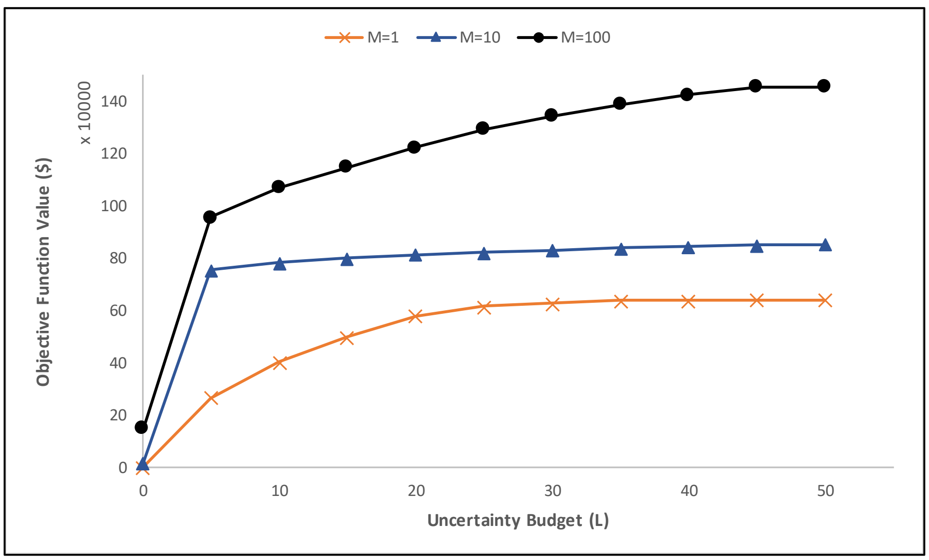

Figure 5: Impact of disutility multipliers and uncertainty budget on OFV

For Gamma robustness, in addition to the impact of the disutility multiplier, one needs to ascertain the role uncertainty budget (measured as L) has on the objective function value. While the disutility multipliers facilitate comparison between different costs in the two stages of **(RRwG)**, the uncertainty budget specifies the interval within which nominal oil-spill probability could vary. Figure 5 depicts the impact of three values of the disutility multipliers on the OFV when the deviation from the nominal probability, i.e. Δp_{jk} , is fixed at 0.2. Close examination of Figure 5 enables us to make the following deductions: *first*, for a given value of the disutility multiplier, larger uncertainty budget will result in higher

OFV; second, for a given value of the uncertainty budget, larger values of the disutility multipliers will yield higher OFV; and *third*, all the three OFV curves are sharply concave, i.e., the increase in OFV with increase in uncertainty budget is rather shallow beyond an uncertainty budget of 20.

Uncertainty Budget	**M=1**		**M=10**		**M=100**	
	1st stage	**2nd stage** *Env.* *Disp.* *Wcc.*	**1st stage**	**2nd stage** *Env.* *Disp.* *Wcc.*	**1st stage**	**2nd stage** *Env.* *Disp.* *Wcc.*
0	0	*2101* *0* *0*	0	*21012* *0* *0*	59000	*86988* *4932* *0*
5	90950	*1648* *0.78* *175836*	678650	*8058* *103* *67870*	762100	*8112* *5385* *180000*
10	186350	*1473* *1.95* *216021*	741650	*6001* *158* *36428*	780350	*5007* *5261* *280150*
15	221000	*1394* *2.58* *276535*	742100	*5744* *175* *52811*	784850	*3874* *5320* *355200*
20	260150	*1317* *3.2* *317492*	751750	*5056* *192* *55799*	784850	*3847* *5321* *430000*
25	374450	*1107* *5.0* *239818*	765250	*3550* *270* *53874*	785300	*3672* *5328* *500000*
30	395600	*1081* *5.2* *231725*	765250	*3550* *270* *61266*	789800	*2675* *5390* *545000*
35	401450	*1068* *5.4* *235211*	765250	*3355* *292* *68900*	789800	*2675* *5390* *590000*
40	410100	*1060* *5.5* *228623*	767950	*2849* *319* *72651*	791150	*2435* *5400* *625380*
45	411000	*1060* *5.4* *229031*	773200	*2411* *342* *74321*	791600	*2393* *5404* *655198*
50	411000	*1060* *5.5* *230044*	777250	*1860* *385* *73972*	791600	*2393* *5404* *685148*

Table 6: Components of OFV ($ '000) using Gamma robustness

Table 6 provides the details of the different cost components for various combinations of disutility multipliers and uncertainty budget. It should be evident that an uncertainty budget of zero will yield the deterministic (or nominal formulation), and hence the result is exactly the same as under **(RRwE)**, i.e., do not open any facility and just pay environment cost for values other than M=100 (Table 5). Furthermore, an increase in uncertainty budget forces more investment in the 1st stage thereby ensuring better coverage, which in turn results in lower environment cost. However, when comparing output of **(RRwG)** to that from **(RRwE)**, we notice relatively higher OFV values under the former. This, we reckon, is an indication of the relatively more aggressive approach of **(RRwG)** in that it recommends higher 1st stage investments. To this point, it is pertinent to mention that for equivalent points (i.e., M=100 and uncertainty budget of 25 in Table 6, and beta of 1 in Table 5: see Appendix A), the worst-case cost of **(RRwG)** is higher than that for **(RRwE)**. In addition, contrasting with (**RRwG**), the worst-case cost is always increasing with the uncertainty budget, and primarily accounts for the increase in the OFV. Thus, it is possible to conclude that Gamma robustness would represent a more risk-averse approach to oil-spill response program.

6.3 Analysis of optimal solutions

In this subsection, we analyze the decoded solutions resulting from the two robust techniques and the nominal formulation. For comparative analysis, we take two steps: first, fix the disutility multiplier at 100 for all the three formulations; second, for equivalent setting as outlined in Appendix A, an uncertainty budget of 25 and $\Delta p_{jk} = 0.2$ is assumed for **(RRwG)**, and a beta value of 1 for **(RRwE)**. Table 7 depicts the resulting optimal solutions, and the pertinent elements. It is interesting that response centers are located in Port of Basque and in St. John's under all formulations, and that the Ellipsoidal robustness yields exactly the same result as the (deterministic or) nominal formulation. However, to account for uncertainty, **(RRwE)** prescribes purchasing a larger number of equipment than does **(SLM)**, and both suggest stockpiling higher number of equipment packages in St. John's. **(RRwG),** on the other hand, calls for opening emergency response facilities at five locations and stockpiling each with almost equal number of equipment, which results in exorbitant cost (see Table 6).

<table>
<tr><th rowspan="2">Formulation</th><th rowspan="2">Locate response facility in:</th><th colspan="2">Number of equipment</th><th rowspan="2">Average Coverage</th></tr>
<tr><th>Acquired</th><th>Dispatched</th></tr>
<tr><td rowspan="5">(RRwG)</td><td>Placentia</td><td colspan="2">344</td><td rowspan="5">83%</td></tr>
<tr><td>Port of Basque</td><td colspan="2">343</td></tr>
<tr><td>St. John's</td><td colspan="2">337</td></tr>
<tr><td>Trepassey</td><td colspan="2">343</td></tr>
<tr><td>Whiffen Head</td><td colspan="2">347</td></tr>
<tr><td rowspan="2">(RRwE)</td><td>Port of Basque</td><td colspan="2">248</td><td rowspan="2">55%</td></tr>
<tr><td>St. John's</td><td colspan="2">848</td></tr>
<tr><td rowspan="2">(SLM)</td><td>Port of Basque</td><td colspan="2">16</td><td rowspan="2">19%</td></tr>
<tr><td>St. John's</td><td colspan="2">104</td></tr>
</table>

Table 7: Optimal robust and nominal solutions

It is clear that the three formulations result in different number of equipment dispatched, which in turn affects the overall coverage. Amongst the three, Gamma robustness formulation results in the dispatch of largest number of equipment thereby ensuring that, on average, 83% of the possible oil-spill profiles are covered. Ellipsoidal robustness formulation ensures an overall coverage of 55%, while the deterministic formulation results in the lowest coverage of 19%. It should be evident that although **(RRwE)** and **(SM)** prescribe opening the same two facilities, the significantly larger number of equipment packages acquired (and dispatched in response to oil-spills) under the former formulation is the cause for the substantially higher coverage.

6.4 Value of robustness

In this subsection, we will provide our analyses from a number of computational experiments in an effort to demonstrate why robust optimization technique is pertinent in this context. To this, we provide an overview of the necessary steps:

- *First*, for a given optimal solution (robust or nominal), the 1st stage decisions are extracted;
- *Second*, several random scenarios for oil-spill probabilities are generated;
- *Third*, for each oil-spill scenario, the 1st stage variables are fixed at the respective values obtained in the first step, while the 2nd stage problem is solved to optimality and the objective function value is recorded; and
- *Fourth*, amongst all the scenarios generated for each (robust or nominal) formulation, the worst-case performance of the 2nd stage problem is recorded.

Table 8 summarizes the results obtained for Gamma robustness, and those for nominal formulation. Under Gamma robustness, for a fixed value of the disutility multiplier (M) at 100, the uncertainty budget (L) was varied between 0 and 65. For each combination of M and L, 400 random scenarios were generated. For comparative narrative an equal number of scenarios were generated for the nominal formulation, and the first two columns, respectively, report the worst-case objective function value (OFV) for the 2nd stage problem and the average of the fourteen worst-case OFV. It should be evident that the uncertainty budget plays no role in the nominal formulation, where the 2nd stage worst-case solution varies between $8.7 million and 9.4 million for an average of $8.96 million. On the other hand, the 4th column depicts the 2nd stage cost for the worst-case scenario for a specific combination of M and L. It is interesting to see that while the OFVs in the 1st column vary randomly, those in the 4th column are decreasing as a function of the uncertainty budget and are meaningfully less than any value reached via nominal formulation of the program. For instance, when the uncertainty budget is 30, the 2nd stage cost is approximately $8.5 million lower under Gamma robustness thereby underscoring the importance of preferring a robust approach. Note that the decrease in 2nd stage cost with higher uncertainty budget is being achieved by accepting larger 1st stage cost, i.e., opening facilities and stockpiling equipment, in the 5th column.

The 6th column labeled V_LT shows the value of the investments made in the robust vis-à-vis nominal solution. More specifically, this column depicts how much cost savings would happen if an additional dollar is invested in the 1st stage of optimal solutions obtained by Gamma formulation. This is obtained by dividing the difference between the average of the 2nd stage worst-case nominal solutions and the 2nd stage worst-case robust solution for the given M and L combination, by the difference between the 1st stage cost of the robust solution for the desired L and the 1st stage cost of the nominal solution. For instance, when L goes from 0 to 5, V_LT is 12.82072 (={8958041-59241}/{762100-59000}). It implies that every additional dollar spent in the 1st stage will result in a cost savings of $12.82072 in the 2nd stage. However, the cost savings show a diminishing impact with an increase in the uncertainty budget, thereby signifying that larger cost savings are more likely under smaller uncertainty budget. Finally, the 7th column labeled V_ST reveals the short-term view on 1st stage investments. More specifically, it is the ratio of the cost savings obtained in the 2nd stage to the extra investments in the 1st stage when uncertainty budget is incremented by 5. Thus, compared to the V_LT values, V_ST has a myopic perspective since it focuses on the 2nd stage gains between two consecutive uncertainty budgets. For example, when L is 30, V_ST is 0.218667 (={8958041-49079}/{789800-785300}). This is an indication of the possible 2nd stage cost savings for every additional dollar spent on the 1st stage investment given the previous uncertainty budget of 25. Note that V_ST is declining more rapidly than V_LT with increase in uncertainty budget, i.e., an additional dollar of investment looks more attractive in relation to the nominal solution, but not as effective if one takes into consideration the last round of investment as reflected via the uncertainty budget. Thus, V_ST is considerably less optimistic than V_LT.

<table>
<tr><th colspan="2">Nominal ($)</th><th colspan="5">Gamma robustness when M=100</th></tr>
<tr><th rowspan="2">Worst: 2nd stage</th><th rowspan="2">Average</th><th rowspan="2">L</th><th colspan="4">($)</th></tr>
<tr><th>Worst: 2nd stage</th><th>1st stage</th><th>V_LT</th><th>V_ST</th></tr>
<tr><td>8795495</td><td rowspan="14">8958041</td><td>0</td><td>8795495</td><td>59000</td><td>NA</td><td>NA</td></tr>
<tr><td>9425305</td><td>5</td><td>59241</td><td>762100</td><td>12.82072</td><td>12.82072</td></tr>
<tr><td>8797929</td><td>10</td><td>51725</td><td>780350</td><td>12.50678</td><td>0.411836</td></tr>
<tr><td>9112949</td><td>15</td><td rowspan="2">50205</td><td rowspan="2">784850</td><td rowspan="2">12.43134</td><td>0.337778</td></tr>
<tr><td>8770021</td><td>20</td><td>NA</td></tr>
<tr><td>8902607</td><td>25</td><td>50063</td><td>785300</td><td>12.42383</td><td>0.315556</td></tr>
<tr><td>8983702</td><td>30</td><td rowspan="2">49079</td><td rowspan="2">789800</td><td rowspan="2">12.34868</td><td>0.218667</td></tr>
<tr><td>8784069</td><td>35</td><td>NA</td></tr>
<tr><td>8781752</td><td>40</td><td>48876</td><td>791100</td><td>12.32618</td><td>0.15037</td></tr>
<tr><td>9295699</td><td>45</td><td rowspan="2">48858</td><td rowspan="2">791600</td><td rowspan="2">12.31864</td><td>0.04000</td></tr>
<tr><td>9076281</td><td>50</td><td>NA</td></tr>
<tr><td>9059621</td><td>55</td><td>48850</td><td>792500</td><td>12.30353</td><td>0.00889</td></tr>
<tr><td>8818463</td><td>60</td><td rowspan="2">48845</td><td rowspan="2">793400</td><td rowspan="2">12.28846</td><td>0.00556</td></tr>
<tr><td>8808676</td><td>65</td><td>NA</td></tr>
</table>

Table 8: Value of Gamma robustness

Table 9 depicts the solutions under Ellipsoidal robustness for different values of beta, and when the disutility multiplier is fixed at 100. In addition, alike Table 8, worst-case solutions and their average resulting from nominal formulation are reported. Examining the solutions in Table 9 enables us to make the following deductions. *First*, alike Table 8, the worst-case 2nd stage costs resulting from robust solutions are much smaller than those from nominal solutions, which underscore the significance of adopting a robust approach to the proposed managerial problem. However, the 2nd stage costs for Ellipsoidal is more than that for Gamma, which is consistent with our earlier observation about the latter being more of a conservative approach, i.e., investing more in the 1st stage so that the emergency network has better response capability (see Tables 5 and 6). *Second*, the 2nd stage cost savings obtained by investing an extra dollar in the 1st stage decision is significantly higher for Ellipsoidal robustness, i.e., potential for reducing worst-case 2nd stage cost is high. *Third*, the gains obtained by more investments in the 1st stage declines under both forms of robustness, however for Ellipsoidal robustness, the decline is more pronounced in the beginning and then stabilizes. *Fourth*, the V_ST values consistently depict a meaningfully higher gains than the equivalent values in Table 8. For instance, V_ST=$5.3697 for a beta of 1, whereas it was only $0.31 for the equivalent uncertainty budget of 25 in Table 8. This happens because the conservative Gamma robustness results in excessive investments in the 1st stage, which undermines the need for any further investments thereby rendering the decisions less effective. On the other hand, Ellipsoidal robustness calls for making 1st stage investments gradually and more cautiously thereby ensuring that subsequent investments remain effective.

Nominal ($)		**Ellipsoidal robustness when M=100**				
Worst: 2nd stage	**Average**	**Beta**	**($) Worst: 2nd stage**	**1st stage**	**V_LT**	**V_ST**
8795495	***9073491***	***0.0***	8795495	59000	NA	NA
8961553		***0.2***	5222543	140000	47.5426	47.5426
9468890		***0.4***	2789268	255600	31.9564	21.0399
8642005		***0.6***	1873593	337100	25.8896	11.2422
9471920		***0.8***	1529419	384800	23.1555	7.21539
9480133		***1.0***	915662	499100	18.5363	5.36970
8891226		***1.2***	496895	578750	16.5014	5.25759
8949046		***1.4***	239346	669650	14.4668	2.83332
9375940		***1.6***	122125	712850	13.6902	2.71345
8944510		***1.8***	83059	738950	13.2222	1.49678
9153238		***2.0***	74173	746600	13.0880	1.16157
8736927		***2.2***	69538	751100	13.0096	1.03000
9103506		***2.4***	65796	758300	12.8810	0.51972
9054479		***2.6***	65031	760550	12.8408	0.34000

Table 9: Value of Ellipsoidal robustness

To sum, both robust approaches demonstrate great potential to prevent exorbitant 2nd stage costs as a result of the uncertain oil-spill probabilities. However, Gamma robustness shows a conservative approach at the beginning thereby obviating the need for further investments, while Ellipsoidal robustness calls for gradual investment in response capability. Hence, Ellipsoidal robustness seems more appropriate since slow and judicious investment in the beginning enables decision makers to adjust resources to make further investments, if required.

6.5 Computational performance

This subsection presents the computational performance of the proposed robust formulations under different settings. All the experiments were obtained using a cut-of-time of 10,000 seconds and were run on a single thread using a laptop with Windows 7, Intel Core i5, 2.5 GHz CPU, and 8 GB RAM. Table 10 depicts the pertinent computation time for both the robust formulations, when the disutility multiplier was fixed at 100.

Ellipsoidal (RRwE)			Gamma (RRwG)	
Beta	**B&B**	**B&C**	**L**	**B&B**
0.0	0.06	0.06	***0***	0.06
0.2	3.20	2.35	***5***	0.11
0.4	6.69	2.50	***10***	0.13
0.6	10.0	3.48	***15***	0.14
0.8	10.5	4.18	***20***	0.16
1.0	18.0	4.60	***25***	0.18
1.2	21.0	5.00	***30***	0.19
1.4	101	5.10	***35***	0.25
1.6	588	5.50	***40***	0.27
1.8	1695	5.80	***45***	0.28
2.0	5551	6.50	***50***	0.30
2.2	-	7.00	***55***	0.31
2.4	-	7.04	***60***	0.33
2.6	-	7.10	***65***	0.34

Table 10: Computational time (in seconds) for robust formulations

For Ellipsoidal robustness, the first approach is branch and bound (B&B) that solves the entire (RRRwE) model through standard branch-and-bound procedure in CPLEX. However, when the size of the problem grows with the increase in the value of beta, the number of second-order cone constraints will increase, which in turn will raise the computational time thereby rendering B&B inefficient. In fact, for instances with beta larger than 2, B&B is unable to reach the optimal solution within the cut-of-time. On the other hand, the proposed branch-and-cut (B&C) procedure proved efficient as it overcame the computational difficulty of the problem and was able to solve instances with large values of beta within a short computational time. It should be mentioned that both B&B and B&C approaches were implemented using C# and CPLEX 12.7 (the default setting), while B&C took advantage of the callback functions. Gamma robustness, on the other hand, could be handled efficiently through standard B&B implemented in CPLEX with relatively small computational time. Also, the increase in computational time with increase in the uncertainty budget is not significant to be a cause of concern, which underlines the fact that modeling uncertainty by Gamma robustness results in relatively simple mixed-integer programming problems.

7. Conclusion

This research studies designing emergency response networks for marine oil-spill incidents, given uncertainty in the the location, size and type of spilled oil. To this end, we propose a robust optimization methodology where uncertainty is modeled through Gamma and Ellipsoidal robustness. The two robust formulations, and the deterministic approach in which the uncertainty is ignored, were tested on realistic data collected from publicly available reports and existing literature.

A comprehensive analysis of the case study based in Newfoundland (Canada) enabled us to make the following conclusions. *First*, in general, the number of response facilities and the consequent equipment stockpile is a function of the trade-off between fixed facility and equipment cost versus expected environmental cost. *Second*, higher values of the disutility multiplier entail larger equipment acquisition at increased cost, which in turn ensures superior coverage to different oil-spill incidents. *Third*, both Gamma and Ellipsoidal formulations yield better coverage than the nominal formulation where uncertainty is ignored, and thus underline the value of robustness. *Fourth*, Gamma robustness adopts a conservative approach thereby obviating the need for further investments, while Ellipsoidal robustness calls for gradual investment in response capability. Thus, the latter seems more appropriate since slow and judicious investments in the beginning enables decision makers to adjust resources to make further investments, if required.

There are a number of directions of future research: *first*, the oil-spill response problem can be considered over a planning horizon that would necessitate decision making over a time-space network and developing an efficient solution technique; and *second*, considering other response measures such as in-situ burning, use of dispersants, etc., in the strategic-tactical oil-spill response planning decisions.

References

Aghajanloo, K., Pirooz, M. D., & Namin, M. M. (2013). Numerical simulation of oil spill behavior in the Persian Gulf. *International Journal of Environmental Research*, 7(1), 81-96.

Baron, O., Milner, J., & Naseraldin, H. (2010). Facility Location: A Robust Optimization Approach. *Production and Operations Management*, *20(5)*, 772-785.

Belardo, S., Harrald, J., Wallace, W.A., & Ward, J. (1984). A partial covering approach to siting response resources to major maritime oil spills. *Management Science*, *30*, 1184-1196.

Ben-Tal, A., El Ghaoui, L., & Nemirovski, A. (2009). *Robust optimization*. Princeton University Press, Princeton, NJ.

Ben-Tal, A., Do Chung, B., Mandala, S.R., & Yao, T. (2011). Robust optimization for emergency logistics planning: Risk mitigation in humanitarian relief supply chains. *Transportation Research Part B - Methodological*, *45*, 1177-1189.

Bertsimas, D., & Sim, M. (2004). Price of robustness. *Operations Research*, *52*, 35-53.

Bertsimas, D., & Thiele, A. (2006). A robust optimization approach to inventory theory. *Operations Research*, *54(1)*, 150-168.

CBC. (2018). Oil vessel off the Newfoundland coast spills 250,000 litres of crude. www.cbc.ca (Accessed: November 17, 2018).

ECRC. (2005). Schedule of Rates: Eastern Canada Response Corporation.

Etkin, S. (1999). Estimating Cleanup Costs for Oil Spills. *1999 International Oil Spill Conference*.

Etkin, S. (2000). Worldwide Analysis of Marine Oil Spill Cleanup Cost Factors. *Arctic and Marine Oil Spill Program Technical Seminar*.

Fischetti, M., Salvagin, D., & Zanette, A. (2009). Fast approaches to improve the robustness of a railway timetable. *Transportation Science*, *43*, 321-335.

French-McCay, D.P. (2004). Oil spill impact modeling: development and validation. *Environmental Toxicological Chemistry*, *23(10)*, 2441-2556.

Gabriel, V., Murat, C., & Thiele, A. (2014). Recent advances in robust optimization: An overview. *European Journal of Operational Research*, *235*, 471-483.

Garrett, R. A., Sharkey, T. C., Grabowski, M., & Wallace, W. A. (2017). Dynamic resource allocation to support oil spill response planning for energy exploration in the Arctic. *European Journal of Operational Research*, 257(1), 272-286.

Gkonis, K.G., Kakalis, N.M., Ventikos, N.P., Ventikos, Y., & Psaraftis, H.N. (2007). A model based approach for tactical decision making in oil spill response. In: *International Symposium on Maritime Safety, Security & Environmental Protection*, Athens, Greece.

Gregory, C., Darby-Dowman, K., & Mitra, G. (2011). Robust optimization and portfolio selection: The cost of robustness. *European Journal of Operational Research*, *212*, 417-428.

Grubesic, T.H., Wei, R., & Nelson, J. (2017). Optimizing oil spill cleanup efforts: A tactical approach and evaluation framework. *Marine Pollution Bulletin*, *125*, 318-329.

Iakovou, E., Chi, M., Douligeris, C., & Korde, A. (1997). Optimal location and capacity of emergency cleanup equipment for oil spill response. *European Journal of Operational Research*, *96*, 72-80.

Kontovas, C.A., Psaraftis, H.N., & Ventikos, N.P. (2010). An empirical analysis of IOPCF oil spill cost data. *Marine Pollution Bulletin*, *60(9)*, 1455-1466.

Louveaux, F.V. (1986). Discrete stochstic location models. *Annals of Operations Research*, *6*, 23-34.

Owen, S.H., & Daskin, M.S. (1998). Strategic facility location: A review. *European Journal of Operational Research*, *111*, 423-447.

Ornitz, B., Champ, M. (2002). *Oil spills first principle: prevention and best response*. Elsevier.

Psaraftis, H.N., Tharakan, G.G., & Ceder, A. (1986). Optimal response to oil spills: the strategic decision case. *Operations Research*, *34*, 203-217.

Psarros, G., Skjong, R., & Vanem, E. (2011). Risk acceptance criterion for tanker oil spill risk reduction measures. *Marine Pollution Bulletin*, *62(1)*, 116-127.

Reed, M., Aamo, O.M., & Daling, P.S. (1995). Quantitative analysis of alternate oil spill response strategies using OSCAR. *Spill Science Technology Bulletin*, *2(1)*, 67-74.

Ruszczynski, A., & Shapiro, A. (2003). A stochastic programming model. In: Ruszczynski, A., Shapiro, A., editors. Stochastic programming: handbooks in operations research and management science, vol. 10. Amsterdam: Elsevier; page 1-63.

Sen, S., & Higle, J.L. (1999). An introductory tutorial on stochastic linear programming models. *Interfaces*, *29*, 33-61.

Snyder, L. (2006). Facility location under uncertainty: a review. *IIE Transactions*, *38*, 537-554.

Srinivasa, A.V., & Wilhelm, W.E. (1997). A procedure for optimizing tactical response in oil spill clean up operations. *European Journal of Operational Research*, *102*, 554-574.

Transport Canada. (2007). Environmental oil spill risk assessment for the south coast of Newfoundland. *Transport Canada Report: TP 14740E*.

Transport Canada. (2010). Assessment of proposals related to oil spill risk for the south coast of Newfoundland. *Transport Canada Report: TP 15039E*.

UNCTAD. (2013). Review of Marine Transport. www.unctad.org. Accessed March 10, 2019.

Vanem, E., Antao, P., Ostvik, I., & de Comas, F.D.C. (2008). Analysing the risk of LNG carrier operations. *Reliability Engineering & System Safety*, *93(9)*, 1328-1344.
Verma, M., Gendreau, M., & Laporte, G. (2013). Optimal location and capability of oil-spill response facilities for the south coast of Newfoundland. *Omega*, *41*, 856-867.
Wilhelm, W.E., & Srinivasa, A.V. (1996). A strategic, area-wide contingency planning model for oil spill cleanup operations with applications to demonstrated to the Galveston Bay Area. *Decision Sciences*, *27*, 767-799.
Wilhelm, W.E., & Srinivasa, A.V. (1997). Prescribing tactical response for oil spill clean up operations. *Management Science*, *43(3)*, 386-402.
You, F., & Leyffer, S. (2011). Mixed-integer dynamic optimization for oil-spill response planning with integration of a dynamic oil weathering model. *AIChE Journal*, *57*, 3555-3564.
Zhong, Z., & You, F. (2011). Oil spill response planning with consideration of physiochemical evolution of the oil slick: a multiobjective optimization approach. *Computers and Chemical Engineering*, *35*, 1614-1630.

Appendix A

We describe a procedure by which an equivalent beta value of Ellipsoidal robustness for each uncertainty budget of Gamma robustness can be calculated.

Let us assume that in Gamma robustness L spill probabilities can each deviate by a fixed value of $\Delta p_{jk} = V$. Then, the beta value associated with Ellipsoidal robustness can be obtained by the following formula:

$$\beta = \sqrt{\sum_{j}\sum_{k}\Delta p_{jk}^2} = \sqrt{(V^2 + V^2 + \cdots \ldots . + V^2)} = \sqrt{LV^2} = V\sqrt{L}$$